



# Mathematics Grade 4 Curriculum

Implemented September 2008

## Acknowledgements

The Department of Education of New Brunswick gratefully acknowledges the contributions of the following groups and individuals toward the development of the *New Brunswick Grade 4 Mathematics Curriculum Guide*:

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education: *The Common Curriculum Framework for K-9 Mathematics*, May 2006. Reproduced (and/or adapted) by permission. All rights reserved.
- Alberta Education (Department of Education)
- Newfoundland and Labrador Department of Education
- Prince Edward Island Department of Education
- The Elementary Mathematics Curriculum Development Advisory Committee
- The Grade 4 Curriculum Development Team:
  - Jeanette Clowater, School District 17
  - Gail Girouard, School District 15
  - Beverly Janes, School District 8
  - Brenda Johnston, School District 18
  - Joan Manuel, School District 10
- Cathy Martin, Learning Specialist, K-8 Mathematics and Science, NB Department of Education
- Mathematics Learning Specialists, Numeracy Leads, and mathematics teachers of New Brunswick who provided invaluable input and feedback throughout the development and implementation of this document.

2008  
Department of Education  
Educational Programs and Services

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## BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, the Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

## BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are

engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

### **GOALS FOR MATHEMATICALLY LITERATE STUDENTS**

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

### **OPPORTUNITIES FOR SUCCESS**

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices. Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals. Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

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## **DIVERSE CULTURAL PERSPECTIVES**

Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of all students.

Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components. Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. It is important to note that these general instructional strategies may not apply to all students.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

## **ADAPTING TO THE NEEDS OF ALL LEARNERS**

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

## **CONNECTIONS ACROSS THE CURRICULUM**

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

## ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

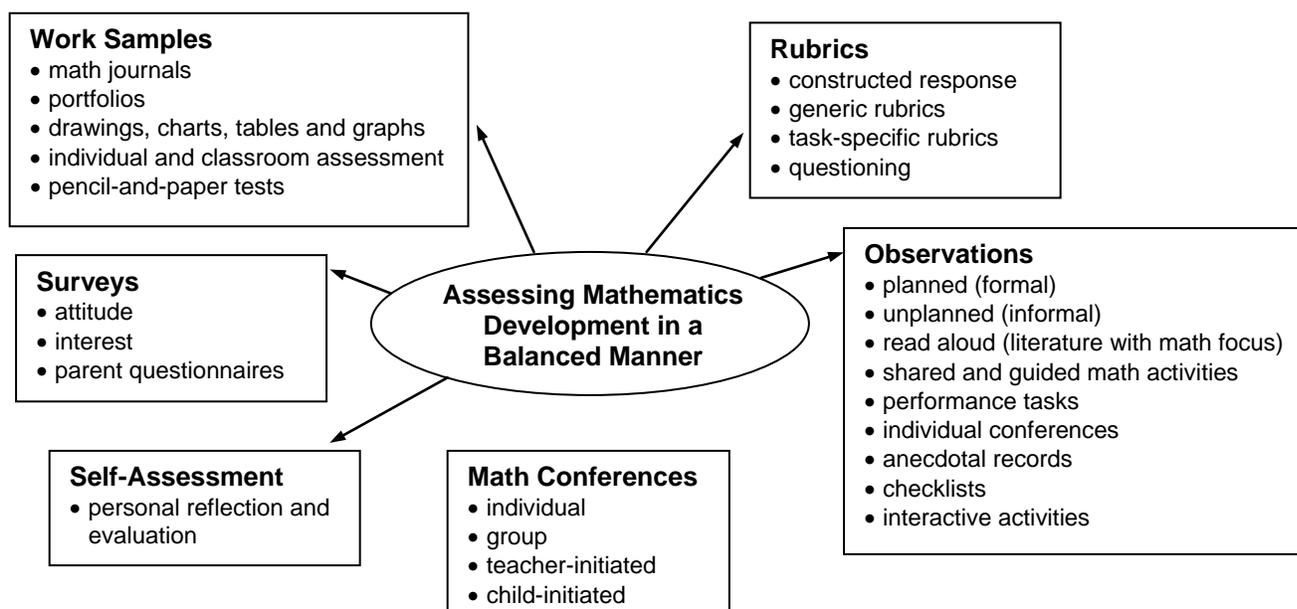
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment*, or assessment of learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

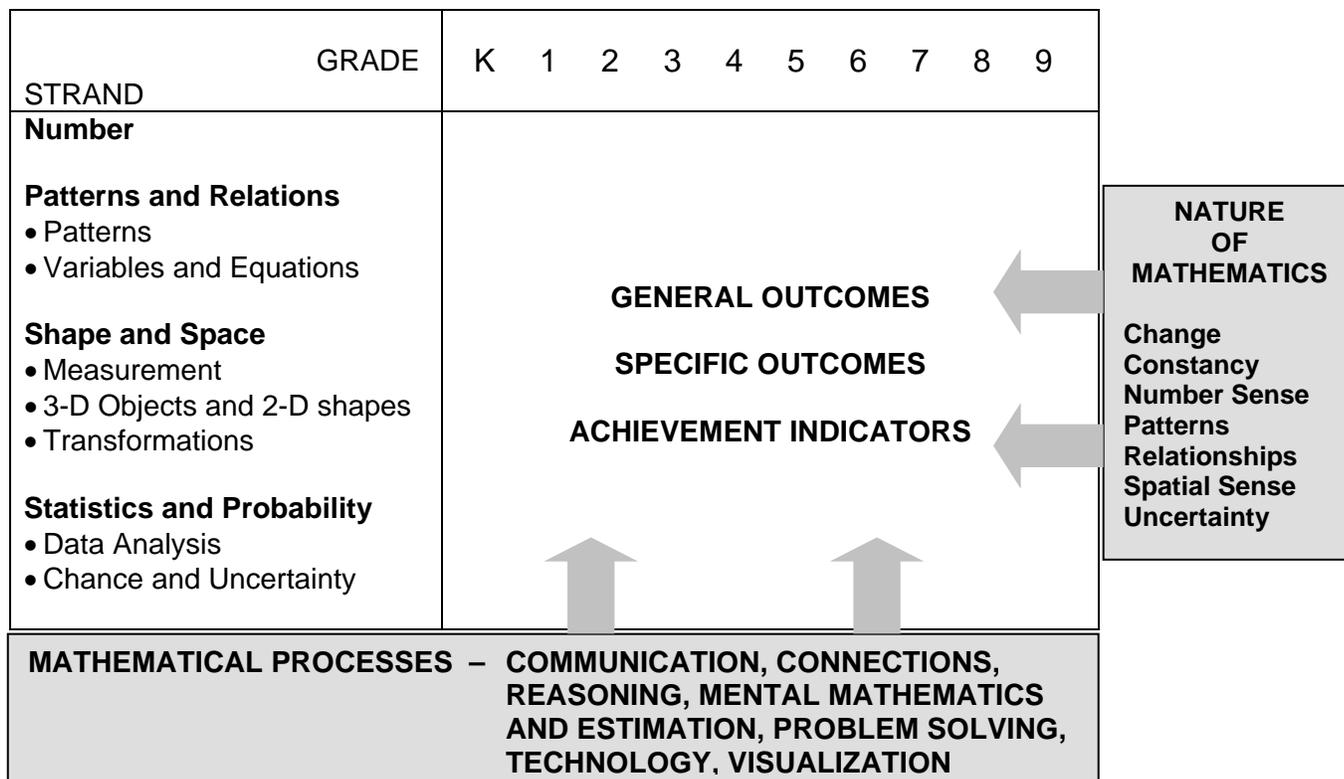
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, *Mathematics Assessment: A practical handbook*, 2001, p.22)



## CONCEPTUAL FRAMEWORK FOR K – 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



### INSTRUCTIONAL FOCUS

The New Brunswick Curriculum is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.
- There is a greater emphasis on mastery of specific curriculum outcomes.

The mathematics curriculum describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between strands.

## MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

### Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

### Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

*“Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p. 5).*

### Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns and test these

generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

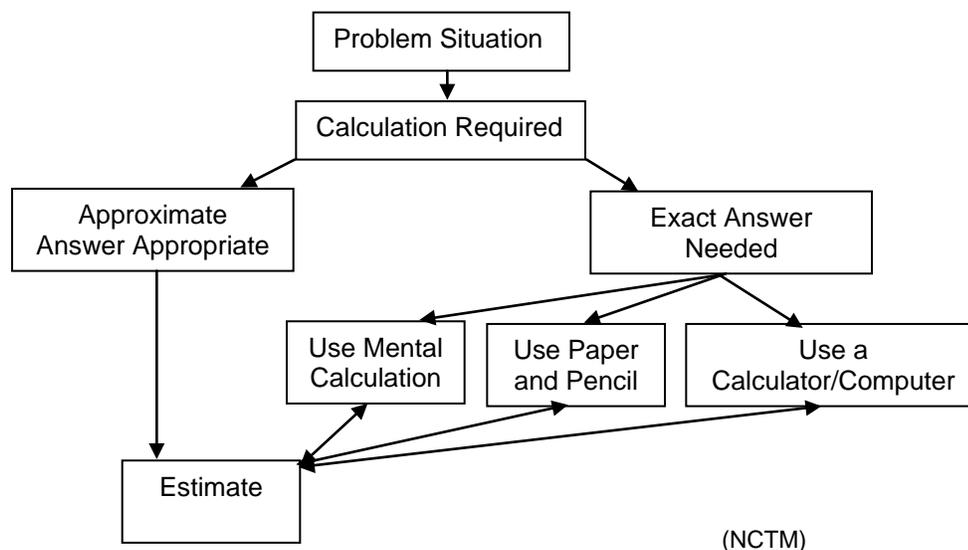
### Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*” (Rubenstein, 2001). Mental mathematics “*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision making process as described below.



### Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “*How would you...?*” or “*How could you...?*” the problem-solving approach is being modeled. Students develop their own problem-solving strategies by being open to listening, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

### **Technology [T]**

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K–3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

### **Visualization [V]**

Visualization “*involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world*” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

## NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change, constancy, number sense, relationships, patterns, spatial sense** and **uncertainty**.

### Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12 ... can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

### Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is  $180^\circ$
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

### Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

### Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

**Patterns**

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

**Spatial Sense**

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four.

**Uncertainty**

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

## STRUCTURE OF THE MATHEMATICS CURRICULUM

### STRANDS

The learning outcomes in the New Brunswick Curriculum are organized into four strands across the grades, K–9. Strands are further subdivided into sub-strands which are the general curriculum outcomes.

### OUTCOMES AND ACHIEVEMENT INDICATORS

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

**General Curriculum Outcomes (GCO)** are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

**Specific Curriculum Outcomes (SCO)** are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

**Achievement Indicators** are one example of a representative list of the depth, breadth and expectations for the outcome. Achievement indicators are pedagogy and context free.

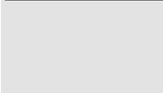
Strand	General Curriculum Outcome (GCO)
<b>Number (N)</b>	<b>Number:</b> Develop number sense
<b>Patterns and Relations (PR)</b>	<b>Patterns:</b> Use patterns to describe the world and solve problems
	<b>Variables and Equations:</b> Represent algebraic expressions in multiple ways
<b>Shape and Space (SS)</b>	<b>Measurement:</b> Use direct and indirect measure to solve problems
	<b>3-D Objects and 2-D Shapes:</b> Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them
	<b>Transformations:</b> Describe and analyze position and motion of objects and shapes
<b>Statistics and Probability (SP)</b>	<b>Data Analysis:</b> Collect, display and analyze data to solve problems
	<b>Chance and Uncertainty:</b> Use experimental or theoretical probabilities to represent and solve problems involving uncertainty

## CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students’ learnings at a particular grade level are part of a bigger picture of concept and skill development.

As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The specific curriculum outcomes (SCOs) are presented on individual four-page spreads as illustrated below.

GCO:
SCO: (specific curriculum outcome and mathematical processes)
Key for mathematical processes
<b>Scope and Sequence</b>
<u>Current Grade</u>

<b>Elaboration</b>
<u>Guiding Questions</u>
(Describes the “big ideas” and what the students should learn this year in regards to this concept.)

Page 1

GCO:
SCO:
<b>Achievement Indicators</b>
<u>Guiding Questions</u>
(Describes what could be observed to determine whether students have met the specific outcome.)

Page 2

GCO:
SCO:
<b>Planning for Instruction</b>
<u>Guiding Questions</u>
<u>Choosing Instructional Strategies</u>
(Lists general strategies to assist in teaching this outcome.)
<u>Suggested Activities</u>
(Lists possible specific activities to assist students in learning this concept.)
<u>Possible Models</u>

Page 3

GCO:
SCO:
<b>Assessment Strategies</b>
<u>Guiding Questions</u>
(Overview of assessment)
<u>Whole Class/Group/Individual Assessment</u>
(Lists sample assessment tasks.)
<b>Follow-up on Assessment</b>
<u>Guiding Questions</u>

Page 4

SCO: <b>N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.</b> [C, CN, V]			
[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<b>N2</b> Represent and describe numbers to 1000, concretely, pictorially and symbolically.	<b>N1</b> Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.	<b>N1</b> Represent and describe whole numbers to 1 000 000.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should recognize the value represented by each digit in a number, as well as what the number means as a whole. The digit “2” in 2300 represents 2 thousands whereas the digit “2” in 3200 represents 2 hundreds.

Students should have many opportunities to:

- **model** numbers containing zeroes. For example, 1003 means 1 thousand and 3 ones;
- **read** numbers several ways. For example, 9347 is read 9 thousand, three hundred forty-seven but might also be expressed as 93 hundred, 47 (other examples may include: 9 thousands, 34 tens, 7 ones; 9 thousands, 33 tens, 17 ones);
- **record** numbers. For example, ask students to **write** twenty-eight hundred sixty; a number which is eighty less than ninety thousand; as well as write numbers in standard form and **expanded notation** ( $7453 = 7000 + 400 + 50 + 3$ ).

Through these experiences, students will develop flexibility in identifying, modeling, and representing numbers up to 10 000. It is also important for students to gain an understanding of the relative size (magnitude) of numbers through real life contexts that are personally meaningful. Use numbers from student’s experiences, such as capacity for local arenas, or population of the school or community. Students can use these **personal referents** to think of other large numbers. **Benchmarks** that students may find helpful are multiples of 100 and 1000, as well as 250, 500, 750, 2500, 5000, and 7500.

Include situations in which students use a variety of models, such as:

- base ten blocks (e.g., to model 10 000 have the class make a long rod with 10 big cubes. It will be a 10 thousand rod. Students should recognize that this also models 10 000 unit cubes. )
- money (e.g., How many \$100 bills are there in \$9347?)
- place value charts, such as shown below.

Thousands			Ones		
H	T	O	H	T	O

The focus of instruction should be on ensuring students develop a strong sense of number.

SCO: N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.  
[C, CN, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Write a given numeral 0 to 10 000 in words.
- Represent a given numeral using a place value chart or diagrams.
- Describe the meaning of each digit in a given numeral.
- Express a given numeral in expanded notation, e.g.,  $4301 = 4000 + 300 + 1$ .
- Write the numeral represented by a given expanded notation. e.g.,  $2000 + 400 + 60 = 2460$ .
- Explain and show the meaning of each digit in a given 4-digit numeral with all digits the same, e.g., for the numeral 2222, the first digit represents two thousands, the second digit two hundreds, the third digit two tens and the fourth digit two ones.
- Write a given numeral using proper spacing without commas. The accepted convention for 4-digit numbers is to not leave a space. For numbers with 5 or more digits, leave a small space between each group of 3 digits starting from the right. (e.g., 4567, 10 000, 287 981). If too large a space is used, the number may be misinterpreted as two separate number.
- Read a given four-digit numeral without using the word “and,” e.g., 5321 is five thousand three hundred twenty one, NOT five thousand three hundred AND twenty one. Note: The word “and” is reserved for reading decimal numbers (e.g., 3.8 is read as “three *and* eight tenths”).

SCO: N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.  
[C, CN, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Invite students to investigate the length of a line comprising 10 000 pennies. Encourage students to share the various strategies they used to investigate this problem. It is also important to have them share strategies that they considered, but rejected and explain their reasoning.
- Use base ten blocks or have students draw pictorial representations of the blocks. Have students use them to explore what numbers might be represented by using exactly 10 base ten blocks. (Note: It is important to use the correct vocabulary when referring to the blocks; “flat”, not “hundred flat” and “rod”, not “ten rod”, etc., so students are flexible in their thinking of the models when working with decimals.)

### **Suggested Activities**

- Provide a stack of four sets of cards numbered 0 - 9. From the stack of 40 cards, have students select 5 cards and arrange them to make the greatest possible number. Ask them to record and read the number and to rearrange the cards to make the least possible number. Have this number recorded under the larger number. As an extension, have the students estimate the difference between the two numbers. This activity is an ideal opportunity for students to practise front-end subtraction (left-to-right calculations).
- Have students, as a class, create a “ten thousands” chart. Provide each small group of students with hundred grids (or other pictorial representations such as arrays of dots) and have them create a model to represent 1000. Combine these models to create a class representation of 10 000.
- Ask students to create a four-digit number using 9, 2, 7, and 5. The digit in the hundreds place must be two more than the ones place. List all of the possible numbers.
- Ask questions about the reasonableness of numbers such as, “Would it be reasonable for an elementary school to have 9600 students?” or “Would it be reasonable for an elevator to hold 20 people?” “Would someone be able to drive twenty-six hundred kilometres in a day?” “Would it be reasonable to pay \$5 000 for a boat/book/computer?” Investigate and discuss possible answers. Have students create their own “reasonable” questions about a variety of topics.
- Have the students find large numbers from newspapers and magazines. Ask them to share and discuss the numbers within their group. Have students read, write, and model the numbers in different ways.

**Possible Models:** base ten blocks, place value charts, money, number lines, hundred grid, number cards, Learning Carpet<sup>®</sup>

SCO: N1: Represent and describe whole numbers to 10 000, concretely, pictorially and symbolically.  
[C, CN, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students to use base ten blocks to model 2016 in three different ways. Have students explain their models.
- Ask students to record a series of numbers that have been read to them (such as eight thousand eighty-two, sixteen hundred five). Include examples such as the greatest 4-digit number or a number one hundred less than the greatest 4-digit number.
- Ask: How are 903 and 9003 different? Similar?
- Tell the student that a boat costs \$6135. Ask: If one were to pay for it in \$100 bills, how many would be needed? Extend by asking how many \$10 bills would be needed.
- Ask the students to write a number that has 980 tens.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N2: Compare and order numbers to 10 000.**  
[C, CN]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<b>N3</b> Compare and order numbers to 1000.	<b>N2</b> Compare and order numbers to 10 000.	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

**Comparing** and **ordering** numbers is fundamental to understanding numbers. Students should investigate meaningful contexts to compare and order two or more numbers, both with and without models. For example, ask them to compare and order populations of communities or capacities of arenas.

Students must realize that when comparing two numbers with the same number of digits, the digit with the greatest value needs to be addressed first. For example, when asked to explain why one number is greater or less than another, they might say that  $2542 < 3653$  because 2542 is less than 3 thousands while 3653 is more than 3 thousands. When comparing 6456 and 6546, students should begin comparing the thousands and then compare each place value to the right.

Students must recognize that when comparing the size of a number, the digit 4 in 4289 has a greater value than the digit 9 and they should be able to provide an explanation.

SCO: N2: Compare and order numbers to 10 000.  
[C, CN]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Order a given set of numbers in ascending or descending order and explain the order by making references to place value.
- Create and order three different 4-digit numerals.
- Identify the missing numbers in an ordered sequence or on a number line.
- Identify incorrectly placed numbers in an ordered sequence or on a number line.

SCO: **N2: Compare and order numbers to 10 000.**  
[C, CN]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

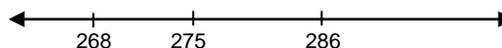
### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide the students with opportunities to practise comparing numbers such as 9098 and 9089 and ask them to explain their reasoning. As an extension, have students order 12 098 and 12 210
- Have students discuss what would be the most appropriate benchmarks for various number lines, such as 0 to 50, 90 to 150, 200 to 1000, or 243 to 2448.
- Provide situations in which students:
  - name numbers which are greater than or less than a given number (Note: In some cases the amount greater or less could be specified, such as 29 more or 3000 less, etc)
  - name numbers which are between given numbers.
- Use a variety of number lines, including open number lines, in which students can place numbers and/or correct numbers that are already placed.



### **Suggested Activities**

- Display a 4-digit number on an overhead calculator (or on a card, on the chalkboard or interactive whiteboard). Have students enter a number which differs by 1 digit on their calculators. Have students read their numbers and ask others to determine if they are greater than or less than the number on the overhead. Collect five, or more, of these numbers and ask the students to order them on a number line. Have students explain how they determined how to order the numbers.
- Assign pairs of students the task of making number cards for their classmates to place in order.
- Provide a list of populations of communities within your area ranging from a few hundred to about ten thousand. Ask students to order them from least to greatest. As an extension students could cluster the populations and graph them (e.g., 0-500, 501-1000, etc., or by 1000s).
- Provide the following riddle for students to solve: "I am thinking of a number. It is between 8000 and 10 000. All the digits are even and the sum of the digits is 16. What are some possibilities?" Use an open number line to display their numbers. Challenge students to write their own riddles.
- Tape number cards on students' backs and ask them to order themselves from least to greatest, without seeing their own number and without talking to each other.
- Have the students find large numbers up to 10 000 from newspapers and magazines and create a collage that would illustrate the order of the numbers from least to greatest.
- Prepare cards for students to order from least to greatest. For example; 6183, 9104, 9080, 7102, 6604, 1999, 6540.
- Ask students to decide which is worth more: 4356 quarters, 8462 dimes or 9999 pennies. Have the students predict first, then use calculators to help solve the problem.
- Use a number line with benchmarks marked on it, and have students place a variety of numbers on the line using the benchmarks as a guide.

**Possible Models:** base ten blocks, number lines, place value charts

SCO: **N2: Compare and order numbers to 10 000.**  
[C, CN]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment

### **Whole Class/Group/Individual Assessment**

- Give the students some number cards and ask them to order them from greatest to least.
- Ask the student how he/she might advise a younger student to determine which of 2 numbers is greater. This could be an interview question or a journal activity.
- Ask the student to record two numbers to meet these requirements: the first has 3 in the thousands place, but is less than the second which has 3 in the hundreds place.
- Ask the students to write a number that would fall about half way between 9490 and 10 000.
- Tell the students that you are thinking of a 4-digit number that has 2 thousands, a greater number of tens, and an even greater number of ones. Ask them to give three possibilities.
- Have students create all of the possible numbers using the digits: 8, 9, 7, 6. Have students place their created numbers on a number line.
- Tell the student that Jodi's number had 9 hundreds, but Fran's had only 6 hundreds. Fran's number was greater. Ask: How was this possible?
- Ask: Which number below must be greater? Explain why.  
4□□2 or 9□□3
- Ask the student how many whole numbers are greater than 8000, but less than 8750.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

<p>SCO: <b>N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:</b></p> <ul style="list-style-type: none"> <li>• <b>using personal strategies for adding and subtracting</b></li> <li>• <b>estimating sums and differences</b></li> <li>• <b>solving problems involving addition and subtraction.</b></li> </ul> <p>[C, CN, ME, PS, R]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>N8</b> Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem solving context.</p> <p><b>N9</b> Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1, 2 and 3-digit numerals) by: using personal strategies for adding and subtracting with and without the support of manipulatives; creating and solving problems in contexts that involve addition and subtraction of numbers concretely, pictorially and symbolically.</p>	<p><b>N3</b> Demonstrate an understanding of addition of numbers with sums to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by:</p> <ul style="list-style-type: none"> <li>• using personal strategies for adding and subtracting</li> <li>• estimating sums and differences</li> <li>• solving problems involving addition and subtraction.</li> </ul>	<p><b>N2</b> Use estimation strategies, including: front-end rounding, compensation, compatible numbers; in problem-solving contexts.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

**Personal strategies** need to make sense to students, be accurate, and efficient. These alternatives are as valid as a traditional algorithm. When introducing addition and subtraction with 3- and 4-digit numbers, it is helpful to use base ten blocks to model the operations concretely, before representing the calculation pictorially and symbolically.

Students need to recognize that **estimation** is a very useful skill in their lives. To be efficient when estimating sums and differences mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Some strategies to consider: using **benchmarks, rounding, front-end addition** and **subtraction (left-to-right calculations)**, and clustering of **compatible numbers**.

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

**Computational fluency** is a balance between **conceptual understanding** (thinking about the structure of numbers and the relationship among numbers and the operations) and **computational proficiency** (includes both efficiency and accuracy) (NCTM, 2000, p. 35).

**SCO: N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction.

[C, CN, ME, PS, R]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Explain how to keep track of digits that have the same place value when adding numbers, limited to 3- and 4-digit numerals.
- Explain how to keep track of digits that have the same place value when subtracting numbers, limited to 3- and 4-digit numerals.
- Represent concretely, pictorially, symbolically the addition and subtraction of whole numbers up 4-digit by 4-digit.
- Describe a situation in which an estimate rather than an exact answer is sufficient.
- Estimate sums and differences using different strategies, i.e., front-end estimation and compensation.
- Solve problems that involve addition and subtraction of whole numbers (one or more steps/where some numbers may be irrelevant). Explain solutions to problems.
- Create a problem given a number sentence for addition or subtraction.
- Solve problems that involve addition and subtraction in more than one way, limited to 3- and 4-digit numerals. For example,  $385 + \square = 500$  or  $500 - 385 = \square$ .

**SCO: N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:**

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction.

[C, CN, ME, PS, R]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Encourage students to estimate prior to doing calculations and/or as a check once they have completed the calculation.
- Use a variety of models, such as base ten blocks and number lines to assist in estimation.
- Provide students with a variety of estimation strategies, including:
  - benchmarks:  $207 - 126$  would give an answer between 75 (200 - 125), and 85 (210 - 125)
  - rounding:  $439 + 52$  is approximately  $440 + 50$ .
  - front-end addition:  $138 + 245 = 370$  (200 + 100 is 300, 30 + 40 is 70 for an estimate of 370). Some students may include the ones in their estimate making their answer 380.
  - front-end subtraction:  $476 - 348 = 130$  (400 - 300 is 100, 70 - 40 is 30, 6 and 8 are about the same so they can be ignored making the estimate 130).
  - clustering: cluster the 29, 35, and 42 together to make 100.
- Use problem solving strategies, such as skip counting on a number line using place value knowledge.
- Explore personal strategies such as *Add Tens, Add Ones, Then Combine* or *Take Extra Tens, Then Add Back* (Van de Walle & Lovin, vol. 2, 2006, p. 109 -111).
- Reinforce proper math vocabulary. "Regrouping" or "trading" is preferred to using terms such as "borrowing" or "carrying" to describe the addition and subtraction process.

### **Suggested Activities**

- Have students paraphrase various story problems to enhance understanding and to recognize which numbers in a problem refer to a part or to a whole. Share solutions.
- Provide the students with an addition number sentence, such as the following:  $328 + 462 = 330 + 460$ . Have them decide if the number sentence is true or false and explain how they know. Remind the students to think of the equal sign as meaning "the same as" so that they are deciding whether the two sides of the equation balance each other.
- Ask the students to find two numbers with a difference of about 150 and a sum of about 500.
- Present the students with problems and have them decide which problems can be answered with an estimate only and which problems require calculation as well as an estimate. For example:
  - *Will a container that holds 2000 mL be large enough to hold 1350 mL of water from another container as well as 1015 mL of water from a different container?*

**Possible Models**: calculators, base ten blocks, place value charts, number lines (including open number lines), balance scale

SCO: N3: Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals) by:

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction.

[C, CN, ME, PS, R]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Model the addition of 1273 and 485 using concrete and/or visual representations and record the process symbolically. Students must be able to explain their method.
- Subtract 248 from 5073 and model the subtraction using concrete or visual representations. Record the process symbolically. Students must be able to explain their method.
- Create an addition or subtraction story problem for the number sentence:  
 $330 - 185 = \square$  or  $185 + \square = 330$ .
- Determine the sum/difference of 3185 and 628 using a personal strategy and explain how the strategy works.
- Present students with the following problem:  
You drink 250 mL of milk on the first day, 375 mL of milk the second day and 450 mL of milk on the third day. About how many millilitres of milk did you drink during these three days? Stimulate the students' thinking by asking whether 900 mL would be a good estimate for the answer.
- Tell the student that Jari said, "To estimate  $583 - 165$ , I think, 575 subtract 175." Ask him/her if the estimate will be high or low, and to explain why Jari might have chosen to estimate in this way.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

<p>SCO: <b>N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.</b> [C, CN, R]</p> <p><b>N5: Describe and apply mental mathematics strategies, such as:</b></p> <ul style="list-style-type: none"> <li>• skip counting from a known fact</li> <li>• using doubling or halving</li> <li>• using doubling or halving and adding or subtracting one more group</li> <li>• using patterns in the 9s facts to determine basic multiplication facts to <math>9 \times 9</math> and related division facts.</li> </ul> <p>[C, CN, ME, PS, R]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>N10</b> Apply mental mathematics strategies and number properties to determine answers for basic addition facts and related subtraction facts (to 18).</p> <p><b>N11</b> Demonstrate an understanding of multiplication to <math>5 \times 5</math></p>	<p><b>N4</b> Explain the properties of 0 and 1 for multiplication and the property of 1 for division.</p> <p><b>N5</b> Describe and apply mental mathematics strategies, such as:</p> <ul style="list-style-type: none"> <li>• skip counting from a known fact</li> <li>• using doubling or halving</li> <li>• using doubling or halving and adding or subtracting one more group</li> <li>• using patterns in the 9s facts</li> <li>• using repeated doubling to determine basic multiplication facts to <math>9 \times 9</math> and related division facts.</li> </ul>	<p><b>N3</b> Apply mental mathematics strategies and number properties, such as: skip counting from a known fact; using doubling or halving; using patterns in the 9s facts; using repeated doubling or halving; to determine answers for basic multiplication facts to 81 and related division facts.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

It is important to address the properties of zero and one in multiplication. A number line can be used to illustrate both of these properties. To explore that the **product** is 0 when multiplying by 0,  $3 \times 0$  can be shown by making 3 hops of 0 or making 0 hops of 3 (Van de Walle & Lovin, vol. 1, 2006, p.85). The property of multiplying and dividing by 1 can similarly be explored on the number line or with an area model. Provide opportunities for students to not only solve multiplication and division problems, but to create their own problems requiring the use of these operations.

Developing basic **multiplication facts** to  $9 \times 9$  and related division facts requires that the students have a strong foundation in patterns, number relationships, place value, and the meaning, relationships and properties of operations as described below:

- patterns are used in developing mental strategies, such as skip counting from a known fact and using the constant sum of the digits in products with the 9s facts;
- number relationships are evident when using the properties of operations or other strategies, such as repeated doubling; e.g.,  $4 \times 6 = (2 \times 6) \times 2 = 24$ ;
- place value is used extensively in various strategies, such as doubling and adding or subtracting one more group; e.g.,  $3 \times 7 = 2 \times 7 + 7 = 14 + 7 = 21$ ;  $9 \times 9 = 10 \times 9 - 9 = 81$ ;
- the meaning of multiplication and division and the connection between the operations is crucial as the students develop understanding of multiplication and division facts. Students who have learned their multiplication facts have automatically learned their division facts.

**Mental math strategies** should be explored in the context of problem solving on an ongoing basis.

**SCO: N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.**  
[C, CN, R]

**N5: Describe and apply mental mathematics strategies, such as:**

- skip counting from a known fact
- using doubling or halving
- using doubling or halving and adding or subtracting one more group
- using patterns in the 9s facts to determine basic multiplication facts to  $9 \times 9$  and related division facts.

[C, CN, ME, PS, R]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

#### **N4**

- Explain the property for determining the answer when multiplying numbers by one.
- Explain the property for determining the answer when multiplying numbers by zero.
- Explain the property for determining the answer when dividing numbers by one.

#### **N5**

- Provide examples for applying mental mathematics strategies:
  - doubling, e.g., for  $4 \times 3$ , think  $2 \times 3 = 6$ , so  $4 \times 3 = 6 + 6$
  - doubling and adding one more group, e.g., for  $3 \times 7$ , think  $2 \times 7 = 14$ , and  $14 + 7 = 21$
  - use ten facts when multiplying by 9, e.g., for  $9 \times 6$ , think  $10 \times 6 = 60$ , and  $60 - 6 = 54$ ; for  $7 \times 9$ , think  $7 \times 10 = 70$ , and  $70 - 7 = 63$
  - halving, e.g., if  $4 \times 6$  is equal to 24, then  $2 \times 6$  is equal to 12
  - relating division to multiplication, e.g., for  $64 \div 8$ , think  $8 \times \square = 64$ .

**SCO: N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.**  
[C, CN, R]

**N5: Describe and apply mental mathematics strategies, such as:**

- skip counting from a known fact
- using doubling or halving
- using doubling or halving and adding or subtracting one more group
- using patterns in the 9s facts to determine basic multiplication facts to  $9 \times 9$  and related division facts.

[C, CN, ME, PS, R]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use various concrete materials and pictorial representations to demonstrate the multiplication and division of zero. For example, use paper plates for the concept of multiplying by zero. Show six plates with zero counters on each. Ask: "How many plates are there?" (six) "How many counters are there on each plate?" (zero) "Six groups of zero are how many?" ( $6 \times 0 = 0$ ).
- Introduce a strategy with the use of materials, practise the strategy, and continue to introduce and practise new strategies. When students have two or more strategies, it is important to focus on strategy selection; choosing the strategy that will be most efficient to determine a particular fact.
- Encourage students to visualize the process for the strategy they are using.  For example,  $4 \times 9 = 4 \times 10$ , or 40; then subtract 4 to equal 36.
- Have students begin with what they know. For example, to figure out  $6 \times 8$ , one student might think, "I know  $5 \times 8 = 40$  and one more 8 is 48." Another might think, "I know  $3 \times 8$  is 24 and twice 24 is 48."
- Use the properties of multiplication in developing mental strategies: the associative property; e.g.,  $(2 \times 2) \times 6 = 2 \times (2 \times 6)$ ; the commutative property:  $3 \times 4$  is read 3 sets, or groups of 4; the product however, is the same if the factors are reversed ( $4 \times 3$ ); distributive property:  $4 \times 8 = (4 \times 5) + (4 \times 3) = 20 + 12 = 32$
- Address the misconception that multiplication always makes the product greater. For example, any number multiplied or divided by 1 remains unchanged.

### Suggested Activities

- Place students in pairs to practise the "double and double again" strategy for facts such as  $4 \times 7$  (e.g.,  $4 \times 7$  is double  $2 \times 7$  which is  $7 + 7$ . Since 2 sevens is 14,  $4 \times 7$  is 28). Students should take turns asking facts and providing answers by repeated doubling.
- Have students play the "Target Game". Create multiplication questions that show a known number multiplied by an unknown number and a "target" they are trying to reach. The goal is to determine the unknown factor that will result in a product closest to the target number, without going over.  
 $5 \times \square \rightarrow 43$  (Target)  $\square$  are left over
- Tell the students that the "6" button on the calculator is not working. Have students suggest ways to solve " $6 \times 64$ " without using this button.

**Possible Models:** arrays, number line (skip counting), area model, counters,  $10 \times 10$  geoboards

**SCO: N4: Explain the properties of 0 and 1 for multiplication and the property of 1 for division.**  
[C, CN, R]

**N5: Describe and apply mental mathematics strategies, such as:**

- skip counting from a known fact
- using doubling or halving
- using doubling or halving and adding or subtracting one more group
- using patterns in the 9s facts to determine basic multiplication facts to  $9 \times 9$  and related division facts.

[C, CN, ME, PS, R]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- What general statement can you make about multiplying any number by one? What general statement can you make about dividing any number by one?
- Create a problem in which you are dividing a number by one.
- Create a problem in which you are multiplying a number by zero.
- Ask a student what general statement they could make about multiplying any number by zero.
- How would you find the answer to the following division fact by relating it to multiplication?  $30 \div 5 =$   
Explain your thinking.
- Ask the student to illustrate two different ways to think about  $6 \times 7$ .
- Ask the student to explain how knowing  $6 \times 5$  helps one to figure out  $12 \times 5$ .
- Ask the student to explain how knowing  $8 \times 10$  helps one to figure out  $8 \times 9$ .
- Ask the student to use counters to show why  $5 \times 8$  is the same as  $3 \times 8 + 2 \times 8$ .
- Tell students that Jasmine said that she was supposed to divide 75 by 5, but found it was easier to divide 150 by 10. Ask them to explain Jasmine's method. Have the students give examples for which it is much easier to double both numbers before dividing.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**

- using personal strategies for multiplication with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products.

[C, CN, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>N11</b> Demonstrate an understanding of multiplication to <math>5 \times 5</math>.</p>	<p><b>N6</b> Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:</p> <ul style="list-style-type: none"> <li>• using personal strategies for multiplication with and without concrete materials</li> <li>• using arrays to represent multiplication</li> <li>• connecting concrete representations to symbolic representations</li> <li>• estimating products.</li> </ul>	<p><b>N2</b> Use estimation strategies, including: front-end rounding; compensation; compatible numbers in problem-solving contexts.</p> <p><b>N5</b> Demonstrate an understanding of multiplication (2-digit by 2-digit) to solve problems.</p>

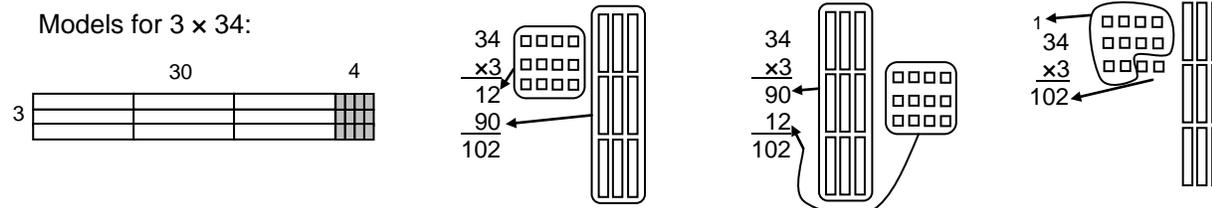
### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Encourage students to **estimate** products and explore their own methods prior to learning the traditional **algorithm** or procedure for finding the **product**. These “personal algorithms” often serve as the procedure of choice, but students need to strive to use the most **efficient strategy** to solve a particular problem.

Students should use a variety of models to investigate multiplication problems to help develop an understanding of the connection between the model and the symbols. It is important to start with a word problem and then have students use materials to determine the product. Base ten blocks serve as a tool for understanding the multiplication operation. It is important that the students use language as they manipulate the materials and record the corresponding symbols for the product. It is not expected that students would be explicitly taught all possible algorithms, but provide opportunities to discover which is most efficient for the numbers included in a given problem. Some examples of possible models for multiplication are shown below.



Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions, preferably choosing topics of interest to them. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

**SCO: N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**

- using personal strategies for multiplication with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products.

[C, CN, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Model a given multiplication problem using the distributive property, e.g.,  $8 \times 365 = (8 \times 300) + (8 \times 60) + (8 \times 5)$ .
- Use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically.
- Create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit.
- Estimate a product using a personal strategy, e.g.,  $2 \times 243$  is close to or a little more than  $2 \times 200$ , or close to or a little less than  $2 \times 250$ .
- Model and solve a given multiplication problem using an array and record the process.
- Solve a given multiplication problem and record the process.

**SCO: N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**

- using personal strategies for multiplication with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products.

[C, CN, ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide regular practice in estimation, accompanied by the sharing of strategies. When assessing estimation, the amount of time provided must be controlled in order to determine whether students are proficient in this skill. The goal is for students to automatically estimate in problem solving situations, not only when instructed to do so.
- Have the students estimate the product to the problem before calculating so that they are better able to determine the reasonableness of their answers.
- Provide a variety of problems representing the different multiplication situations with varying degrees of difficulty to differentiate instruction.
- Provide time for students to create their personal strategies to solve the problem and share these strategies with members of their group or with the entire class.
- Challenge the students to solve the problem another way, do a similar problem without models or clarify the explanation of their personal strategies.

### **Suggested Activities:**

- Ask students how they would use the front-end mental multiplication strategy for questions such as  $3 \times 125 = 375$  ( $3 \times 100 + 3 \times 20 + 3 \times 5$ ) and encourage strategies such as ( $3 \times 100 + 3 \times 25$ ).
- Ask the students to fill in the blanks with 3, 4, and 5 in three different ways and find all the possible products.  $\square \square \times \square$
- Provide students with problems to solve, such as:
  - You travel 375 km each day for 3 days. Will you reach the cabin that is 1200 km away by the end of the third day?
  - You set up 6 rows of chairs with 28 chairs in each row in the gym. Are there enough chairs to seat 180 people? How many chairs did you set up?
  - A kangaroo jumps 135 cm on the first jump and twice as far on the second jump. About how far does it jump in all?
  - You jog for 175 minutes each week. How many minutes do you jog in 28 days?
- Have students use a supermarket flyer. Ask them to select 6 of one item, 4 of another and 10 of a third item and give an estimate for the total.

**Possible Models:** base ten blocks, arrays, area model

**SCO: N6: Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by:**

- using personal strategies for multiplication with and without concrete materials
- using arrays to represent multiplication
- connecting concrete representations to symbolic representations
- estimating products.

[C, CN, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Tell the student that he/she has \$60. Ask: “Do you have enough money to buy 3 CDs if each costs \$17? How do you know?”
- Ask the students if they can reach a cottage that is 1200 km away if they travel 375 km each day for 3 days. Have them explain their thinking.
- Write all the possible number sentences that are represented in the following array. Explain how each number sentence relates to the array.

```
*****
*****
*****
```

- Ask students to model  $24 \times 6$ . Have them explain their model.
- Tell students that for a school assembly, 9 rows of 38 chairs have been placed in the gym. Are there enough chairs for 370 students? Have them explain their thinking.
- Ask students to create and solve a realistic problem that includes the factors 6 and 329.
- Have students solve problems, such as: You save 6 times as much money this year as you saved last year. If you saved \$125 last year, how much money did you save this year?

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

**SCO: N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**

- using personal strategies for dividing with and without concrete materials
- estimating quotients
- relating division to multiplication.

[C, CN, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<b>N12</b> Demonstrate an understanding of division (limited to division related to multiplication facts up to $5 \times 5$ )	<b>N7</b> Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: <ul style="list-style-type: none"> <li>• using personal strategies for dividing with and without concrete materials</li> <li>• estimating quotients</li> <li>• relating division to multiplication.</li> </ul>	<b>N6</b> Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit) and interpret remainders to solve problems.

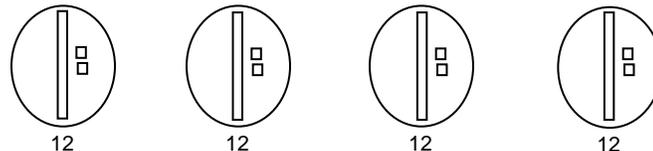
### ELABORATION

#### Guiding Questions:

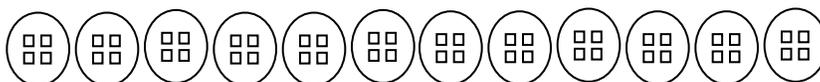
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

The concept of division needs to be taught in conjunction with multiplication. Students need to know the two meanings for division. These are as follows:

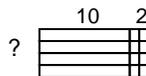
- identifying how many in each group (partitioning or sharing):  
48 pencils in 4 containers (base ten blocks as models)



- identifying how many groups (repeated subtraction): 48 pencils, 4 in each container.



Students can also use the area model with a missing dimension to show the relationship with multiplication.



Students should understand that the **remainder** (the number of units left over) must be less than the **divisor**. Models help to clarify this idea. In grade four, students are expected to express remainders as a digit and not as a fraction or decimal (e.g., a remainder of 7 is written as R7). Students also need to know that the answer for a division sentence is the **quotient** and the number to be divided is the **dividend**.

$$\begin{array}{l} \text{dividend} \rightarrow 138 \div 6 = 23 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow \end{array}$$

Students should have many opportunities to solve and create word problems for the purpose of answering real-life questions of personal interest. These opportunities provide students with a chance to practise their computational skills and clarify their mathematical thinking.

SCO: N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:

- using personal strategies for dividing with and without concrete materials
- estimating quotients
- relating division to multiplication.

[C, CN, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Solve a given division problem without a remainder using arrays or base ten materials.
- Solve a given division problem with a remainder using arrays or base ten materials.
- Solve a given division problem using a personal strategy and record the process.
- Create and solve a word problem involving a 1- or 2-digit dividend.
- Estimate a quotient using a personal strategy, e.g.,  $86 \div 4$  is close to  $80 \div 4$  or close to  $80 \div 5$ .

(It is not intended that remainders be expressed as decimals or fractions.)

**SCO: N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:**

- using personal strategies for dividing with and without concrete materials
- estimating quotients
- relating division to multiplication.

[C, CN, ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ensure students explore the relationship between multiplication and division.
- Provide regular practice in estimation, accompanied by the sharing of strategies.
- Present division questions in context to identify either the sharing (how many in each group: partitioning) or the repeated subtraction (how many groups) meaning.
- Provide a variety of problem structures that include both of the meanings of division used in a real-life context.
- Have students create and share problems that include both of the meanings of division. It is helpful for students to explore a variety of models for solving division questions.

### **Suggested Activities**

- Ask students to use a model to explain to a classmate how to share 86 marbles among five people. Discuss the different strategies used.
- Ask students to make up division problems about situations in the classroom and post them. Encourage them to give examples of both the partition and the repeated subtraction meanings of division. Invite others to try to guess what the division situations are. For example,  $25 \div 6$  (classmates divided into groups of 6. How many groups?).
- Provide a list of division questions to pairs of students and ask them to estimate a quotient and explain their strategy to their partner and tell whether the estimate is too high or too low and why
- Present the students with a problem and have them choose which of the number sentences provided could be used to solve the problem and why they chose it. For example, Dave earned \$96 this month by doing odd jobs for the neighbours. Last month, he earned \$8. How many times more did he earn this month as last month?

$$\begin{array}{lll}
 96 \times 8 = \square & \square = 8 \times 96 & 8 \times \square = 96 \\
 96 \times \square = 8 & 96 \div 8 = \square & 8 \div 96 = \square \\
 \square \div 8 = 96 & 96 \div \square = 8 & 8 \div \square = 96
 \end{array}$$

- Present students with a variety of problems to solve, such as:
  - There are 77 baseball cards to be shared between 2 students. Ask them how they know that there will be a remainder. What about sharing them among 5 students? 7 students?
  - Tyra rode her bicycle every day for 8 days. She cycled 68 km in total. About how far did she ride each day?
- Use base ten blocks to solve the following problem: If the area of a rectangular field is  $182 \text{ m}^2$  and the length is 14 m, how wide is the field?

**Possible Models:** arrays, base ten blocks, counters, money

SCO: N7: Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by:

- using personal strategies for dividing with and without concrete materials
- estimating quotients
- relating division to multiplication.

[C, CN, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Have the students use/draw models to show  $83 \div 3$  and explain their thinking.
- Ask: How many digits are there in the quotient of  $4 \overline{)57}$ ? Explain how you know.
- Present the student with the following problem: You have 72 marbles to share equally among 4 friends. How many marbles will each friend receive? Explain how you know.
- Ask the student to explain the connection between multiplication and division by using counters or base ten blocks. If necessary, suggest to the student to make an array.
- Have the student estimate  $93 \div 5$  and tell whether the estimate is probably high or low and why. Ask the student to suggest another division question for which the same estimate would be appropriate.
- Provide the students with a set of base ten blocks. Ask the student to model 3 different division questions of his/her choice and to write the division sentence for each.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- name and record fractions for the parts of a whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used.

[C, CN, PS, R, V]

[C] Communication

[PS] Problem Solving

[CN] Connections

[ME] Mental Math

[T] Technology

[V] Visualization

[R] Reasoning

and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>N13</b> Demonstrate an understanding of fractions by: explaining that a fraction represents a part of a whole; describing situations in which fractions are used; comparing fractions of the same whole with like denominators.</p>	<p><b>N8</b> Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:</p> <ul style="list-style-type: none"> <li>• name and record fractions for the parts of a whole or a set</li> <li>• compare and order fractions</li> <li>• model and explain that for different wholes, two identical fractions may not represent the same quantity</li> <li>• provide examples of where fractions are used.</li> </ul>	<p><b>N7</b> Demonstrate an understanding of fractions by using concrete and pictorial representations to: create sets of equivalent fractions; compare fractions with like and unlike denominators.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In order for students to construct a firm foundation for fraction concepts, they need to experience and discuss activities that promote the following understandings:

- Fractional parts are equal shares or equal-sized portions of a whole or unit.
- A unit can be an object or a collection of things. More abstractly, the unit is counted as 1. On the number line, the distance from 0 to 1 is the unit.
- Fractional parts have special names that tell how many parts of that size are needed to make the whole. For example, thirds require three parts to make a whole.
- The more fractional parts used to make a whole, the smaller the parts. For example, eighths are smaller than fifths.
- The **denominator** of a fraction indicates by what number the whole has been divided in order to produce the type of part under consideration. Thus, the denominator is a divisor. In practical terms, the denominator names the kind of fractional part that is under consideration. The **numerator** of a fraction counts or tells how many of the fraction parts (or the type indicated by the denominator) are under consideration. Therefore, the numerator is a multiplier—it indicates a multiple of the given fractional part (Van de Walle & Lovin, vol. 1, 2006, p. 251).

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{parts being considered}}{\text{total number of parts in the whole}}$$

Presenting fractions in context will make them more meaningful to students. It is important that students develop visual images for fractions and be able to tell “about how much” a particular fraction represents and learn common benchmarks, such as one half. Students should model fractions using a variety of materials. To strengthen their fraction number sense, it is also recommended that students explore fractions with models where the size of the whole is not the same. In grade four, the focus is on students developing a firm understanding of fractions less than one.

**SCO: N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- name and record fractions for the parts of a whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used.

[C, CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Represent a given fraction using concrete materials.
- Identify a fraction from its given concrete representation.
- Name and record the shaded and non-shaded parts of a given set.
- Name and record the shaded and non-shaded parts of a given whole.
- Represent a given fraction pictorially by shading parts of a given set.
- Represent a given fraction pictorially by shading parts of a given whole.
- Explain how denominators can be used to compare two given unit fractions with numerator 1.
- Order a given set of fractions that have the same numerator and explain the ordering.
- Order a given set of fractions that have the same denominator and explain the ordering.
- Identify which of the benchmarks: 0,  $\frac{1}{2}$ , or 1 is closer to a given fraction.
- Name fractions between two given benchmarks on a number line.
- Order a given set of fractions by placing them on a number line with given benchmarks.
- Provide examples of when two identical fractions may not represent the same quantity, e.g., half of a large apple is not equivalent to half of a small apple; half of ten oranges is not equivalent to half of sixteen oranges.
- Provide an example of a fraction that represents part of a set and, a fraction that represents part of a whole from everyday contexts.

**SCO: N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:**

- name and record fractions for the parts of a whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used.

[C, CN, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Present three models for fractions: part of a region; part of a set; part of a length or measures.
- Ensure students develop an understanding that a fraction is not meaningful without knowing what the “whole” that it is part of.
- Develop conceptual understanding of comparing fractions through a variety of ways to relate fractions including the following:
  - more of the same size in which the denominators of the fractions are the same; e.g., five-eighths is greater than three-eighths;
  - same number of parts but parts of different sizes in which the numerators of the fractions are the same; e.g., three-quarters is greater than three-fifths;
  - more or less than one-half or one whole in which numerator of the fraction is compared to the denominator in deciding its relation to a given benchmark; e.g., three-eighths is less than one-half because three is less than half of eight (Van de Walle & Lovin, vol. 1, 2006, p. 265).
- Use a horizontal line when writing fractions, instead of a slash: e.g.,  $\frac{3}{4}$  not  $\frac{3}{4}$ .

### **Suggested Activities**

- Have the students order a set of fractions. Use sticky notes and put a fraction on each forehead of a group of students (about 4 to 8). The students need to place themselves in order without talking.
- Have students determine what fraction of the letters in their names (or other words) are vowels.
- Have students explore fraction relationships among pattern blocks, fraction pieces, Cuisenaire rods<sup>®</sup>, and other materials.
- Show examples and non-examples of specified fractional parts. Have students identify the wholes that are correctly divided into requested fractional parts and those that are not. For each response, have students explain their reasoning. This activity should be done with a variety of models.
- Tell the student that you have 8 coins. Half of them are pennies. More than one-eighth of them are quarters. The others are nickels. Have the student use coins to represent the situation. How much money might you have? Ask students to create other coin problems using proper fraction notation.
- Provide students with different sizes and shapes of paper and have them estimate and then tear-off different fractional parts, such as one-fifth. Have them explain their thinking. Students can compare their “fifths” as the size of these will vary depending on the size of the whole.

**Possible Models:** number lines, fraction pieces, Cuisenaire rods<sup>®</sup>, hundred grids, hundredths circles, 10 x10 geoboards, metre sticks, money, pattern blocks, grid paper, colour tiles, egg cartons

SCO: N8: Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to:

- name and record fractions for the parts of a whole or a set
- compare and order fractions
- model and explain that for different wholes, two identical fractions may not represent the same quantity
- provide examples of where fractions are used.

[C, CN, PS, R, V]

## ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

### Guiding Questions

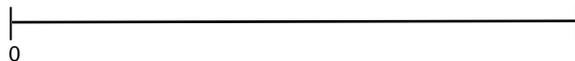
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### Whole Class/Group/Individual Assessment

- Have students place the following fractions on the number line below and verify their positions using models.

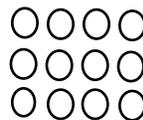
$$\frac{1}{2}, \frac{1}{4}, \frac{1}{10}, \frac{3}{4}, \frac{1}{3}, \frac{7}{8}, \frac{4}{6}$$



- Present the following problem to students: Kiri ate  $\frac{1}{4}$  of her pizza and David ate  $\frac{3}{4}$  of his pizza. Kiri said that she ate more pizza than David. Explain how Kiri could be right by using diagrams and words.
- Place the following pairs of fractions before the student, one at a time. Tell the student to circle the larger fraction and explain in words how he or she knows that the fraction is larger. Then, have them select a manipulative and model the fractions to verify their selection.

$$\frac{1}{5}, \frac{3}{5}, \quad \frac{3}{8}, \frac{3}{5}, \quad \frac{1}{3}, \frac{1}{4}, \quad \frac{4}{8}, \frac{3}{6}, \quad \frac{3}{4}, \frac{9}{10}$$

- Ask the student to tell why, whenever you see a representation of  $\frac{1}{3}$ , there is always a  $\frac{2}{3}$  associated with it.
- Ask the student to colour  $\frac{1}{4}$  of the set of circles.



## FOLLOW-UP ON ASSESSMENT

### Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

<p>SCO: <b>N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.</b> [C, CN, R, V]</p> <p><b>N10: Relate decimals to fractions (to hundredths).</b> [CN, R, V]</p>			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
	<p><b>N9</b> Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.</p> <p><b>N10</b> Relate decimals to fractions (to hundredths).</p>	<p><b>N8</b> Describe and represent decimals (tenths, hundredths, thousandths) concretely, pictorially and symbolically.</p> <p><b>N9</b> Relate decimals to fractions and fractions to decimals (to thousandths).</p> <p><b>N10</b> Compare and order decimals (to thousandths), by using: benchmarks; place value; equivalent decimals.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Number sense with fractions and decimals requires that the students develop a conceptual understanding of fractions and decimals as numbers. To work effectively with fractions and decimals, the students should demonstrate the ability to:

- represent numbers using words, models, diagrams and symbols and make connections among various representations;
- generate equivalent forms of decimals and fraction numbers, such as 0.25 is also one-fourth;
- describe the relative magnitude of numbers by comparing them to common benchmarks, given simple estimates, ordering a set of number, and finding a number between two numbers.

Conceptual understanding of decimals requires that students connect decimals to **whole numbers** and to fractions. Decimals are shown as an extension of the whole number system by introducing a new place value, the **tenth's** place, to the right of the one's place. The tenth's place follows the pattern of the base ten number system by iterating one-tenth ten times to make one whole or a unit (Wheatley & Abshire, 2002, p. 152). Similarly, the **hundredth's** place to the right of the tenth's place iterates one-hundredth ten times to make one-tenth.

Foster an understanding of decimals by ensuring that they are read correctly. For example, 3.4 should be read as 3 and 4 tenths, not 3 point 4, or 3 decimal 4. It is also important that students understand the relationship between fractions and decimals. For example, 12.56 is read as 12 and 56 hundredths. Saying decimal numbers correctly will assist students in gaining an understanding of how decimals relate to fractions. By saying 12 and 56 hundredths, 56 is the numerator and 100 is the denominator. Saying the number correctly also reinforces the idea that the digits to the right of the decimal are part of the whole number.

The connection between decimals and fractions is developed conceptually when the students read decimals as fractions and represent them using the same visuals. For example, 0.8 is read as eight-tenths and can be represented using fraction strips or decimal strips (Wheatley and Abshire, 2002). Students should use a variety of materials to model and interpret decimal tenths and hundredths.

SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**

[C, CN, R, V]

**N10: Relate decimals to fractions (to hundredths).**

[CN, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

### **N9**

- Write the decimal for a given concrete or pictorial representation of part of a set, part of a region or part of a unit of measure.
- Represent a given decimal using concrete materials or a pictorial representation.
- Explain the meaning of each digit in a given decimal with all digits the same.
- Represent a given decimal using money values (dimes and pennies).
- Record a given money value using decimals.
- Provide examples of everyday contexts in which tenths and hundredths are used.
- Model, using manipulatives or pictures, which a given tenth can be expressed as hundredths, e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies.

### **N10**

- Read decimals as fractions, e.g., 0.5 is zero and five tenths.
- Express orally and in written form a given decimal in fractional form.
- Express orally and in written form a given fraction with a denominator of 10 or 100 as a decimal.
- Express a given pictorial or concrete representation as a fraction or decimal, e.g., 15 shaded squares on a hundred grid can be expressed as 0.15 or  $\frac{15}{100}$ .
- Express orally and in written form the decimal equivalent for a given fraction, e.g.,  $\frac{50}{100}$  can be expressed as 0.50.

SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**

[C, CN, R, V]

**N10: Relate decimals to fractions (to hundredths).**

[CN, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

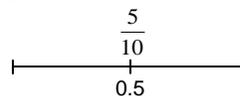
### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ensure students have an understanding of the meaning of decimal numbers. When discussing the meaning of numbers such as 1.1, attention should be given to the multiple meanings of the number (e.g., one whole and one tenth; 11 tenths).
- Help students extend the place-value system to decimals by focusing on the basic pattern of ten. Remind students that 10 ones make 1 ten, 10 tens make 1 hundred, etc. Then, extend this pattern to help students understand that it takes 10 equal parts (tenths) to make 1 whole and 100 equal parts (hundredths) to make 1 whole. Students should recognize that the place value of the digits to the right of the one's place are tenths and hundredths.
- Investigate the relationship between 1.0, 0.1, and 0.01 by making analogies and using real-life objects which are sized proportionally.
- Use a double number line to show equivalence between decimals and fractions.



### **Suggested Activities**

- Use a variety of materials to model number with decimals to the hundredths. Ensure that some models show equivalent fractions/decimals. Shading in 2 tenths of a 100 grid represents the equivalent fraction/decimal of 20 hundredths. Provide students with ample opportunities to write the decimal and the fraction of what the model represents.
- Show students how to make a calculator “count” by ones by pressing +, 1, =, =, ... Next have students press +, 0.1, =, =, ... when the display reaches 0.9, stop and discuss what this means and what the display will look like with the next press. Many students will predict 0.10 (thinking that 10 comes after 9). When the tenth press produces a display of 1 (note: calculators never display trailing zeroes), the discussion should revolve around regrouping 10 tenths for a whole. How many presses to get from one whole number to the next? Repeat counting by 0.01 (Van de Walle & Lovin, vol. 2, 2006, p. 187).
- Ask students to show 2 tenths, if:
  - a large base ten cube represents one whole;
  - a flat represents one whole;
  - a rod represents one whole.
 Extend this to explore hundredths.

**Possible Models:** base ten blocks, hundred grid, metre stick, 10 x 10 geoboard, number lines (including double number lines), hundredths circles, money

SCO: **N9: Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.**

[C, CN, R, V]

**N10: Relate decimals to fractions (to hundredths).**

[CN, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Have students use a hundred grid to shade a capital "T" that takes up more than 0.20 of the grid and one that takes less than 0.20 of the grid. Express the shaded and un-shaded areas as fractions.
- Ask students where they would find decimal numbers in their daily lives.
- Ask the student to use a model of choice to explain why 0.40 and 0.4 are equivalent.
- Provide the student with a number, such as 3.94 and ask the student to:
  - give the number that is 0.1 more than;
  - 1 less than;
  - 0.01 more than.
- Explain to the student that someone forgot to put the decimal in the number 1427. Ask where it could be if the number is less than 100.
- Ask students to read decimal numbers orally. For example: 2.5, 26.9, \$127.60, 44.09, 0.02.
- Have students write the numbers that you say to them. For example: three and two tenths, eighty-seven and six hundredths, fourteen hundredths, five dollars and forty cents, eleven tenths.
- Plot common fraction and decimal equivalents on a number line. For example: one half and five tenths; one fourth and twenty-five hundredths; three-quarters and seventy-five hundredths; one tenth.
- Have students count forward and backward from any number. For example, count on in tenths from 4.7 or count backwards in hundredths from 4.05.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**

- using compatible numbers
- estimating sums and differences
- using mental math strategies to solve problems.

[C, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>N6</b> Describe and apply mental mathematics strategies for adding two 2-digit numerals.</p> <p><b>N7</b> Describe and apply mental mathematics strategies for subtracting 2-digit numerals.</p>	<p><b>N11</b> Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:</p> <ul style="list-style-type: none"> <li>• using compatible numbers</li> <li>• estimating sums and differences</li> <li>• using mental math strategies to solve problems.</li> </ul>	<p><b>N10</b> Relate decimals to fractions and fractions to decimals (to thousandths).</p> <p><b>N11</b> Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).</p>

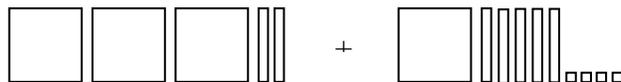
### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

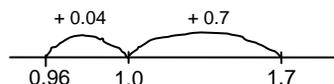
It is essential that students recognize that all of the properties and strategies established for the addition and subtraction of whole numbers also apply to decimal numbers. Students should recognize that adding or subtracting **tenths** (e.g., 3 tenths and 4 tenths are 7 tenths) is similar to adding or subtracting quantities of other items (e.g., 3 apples and 4 apples are 7 apples). The same is true with **hundredths**. Rather than simply telling students to line up decimals vertically, or suggesting that they “add zeroes,” they should be directed to think about what each **digit** represents and what parts go together. For example:  $1.62 + 0.3$ , a student might think, 1 whole, 9 (6 + 3) tenths and 2 hundredths, or 1.92.

Base ten blocks and hundredths grids are useful models to explore these concepts. If a flat represents one whole unit, then  $3.2 + 1.54$  would be modelled as:



These amounts could also be represented by shading hundredths grids (three whole grids and two tenths of a fourth one and one whole grid and fifty-four hundredths of a second one).

Number lines and open number lines can also be useful to model addition and subtraction. For example, to find the answer for  $1.7 - 0.96$ :



Students need to recognize that **estimation** is a useful skill in their lives. To be efficient when estimating **sums** and **differences** mentally, students must be able to access a strategy quickly and they need a variety from which to choose. Situations must be provided regularly to ensure that students have sufficient practice with mental math strategies and that they use their skills as required. When a problem requires an exact answer, students should first determine if they are able to calculate it mentally; this should be determined every time a calculation is required.

**SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:**

- using compatible numbers
- estimating sums and differences
- using mental math strategies to solve problems.

[C, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Predict sums and differences of decimals using estimation strategies.
- Solve problems, including money and measurement, which involve addition and subtraction of decimals, limited to hundredths.
- Ask students to determine which problems do not require an exact solution.
- Determine the approximate solution of a given problem using compatible numbers.
- Determine an exact solution using mental computation strategies.
- Count back change for a given purchase.

SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:

- using compatible numbers
- estimating sums and differences
- using mental math strategies to solve problems.

[C, ME, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Encourage students to estimate prior to calculating answers.
- Use a variety of appropriate models, such as base ten blocks and number lines to assist students in their initial consideration of estimation.
- Use **estimation strategies** including: Compatible numbers: e.g.,  $0.72 + 0.23$  are close to  $0.75$  and  $0.25$  which are compatible numbers so the sum of the decimal numbers must be close to 1. Front-end addition: e.g.,  $32.3 + 24.5 + 14.1$ ; a student might think " $30 + 20 + 10$  is 60 and the ones and tenths clustered together make about another 10 for a total of 70." Front-end subtraction: e.g.,  $1.92 - 0.7$ ; a student might think " $19$  tenths -  $7$  tenths is  $12$  tenths and 2 hundredths more is  $1.22$ ." Rounding: e.g.,  $4.39 + 5.2$  is approximately  $4 + 5$  for an estimate of 9.
- Use **mental computation strategies** include: Compatible numbers: e.g.,  $3.55 + 6.45$  or \$3 and \$6 would be \$9 while 55 cents and 45 cents would make another dollar, for a sum of \$10 or 10. Front-end strategy: e.g.,  $7.69 - 2.45$  A student might think " $7$  ones subtract 2 ones is 5 ones, 6 tenths subtract 4 tenths is 2 tenths and 9 hundredths subtract 5 hundredths is 4 hundredths, so the difference would be 5.24" Compensate: e.g.,  $\$4.99 + 1.98 + 0.99$  could be calculated by finding the sum of  $5 + 2 + 1$  and then subtracting 0.04 or 4 cents. The sum would be \$7.96. Counting on/ counting back:  $\$2 - 1.48$ ; a student might think, "2 more pennies would make \$1.50 and 50 cents more makes \$2 so the difference (change) is 52 cents; Renaming: think of  $3.2 + 0.9$  as 32 tenths + 9 tenths.

### **Suggested Activities**

- Give students word problems that require the addition and/or subtraction of whole numbers and decimals. Particularly appropriate contexts are money and measurement (e.g.,  $3.45 \text{ m} + 721.6 \text{ m}$ ;  $12.4 \text{ kg} - 7.25 \text{ kg}$ ).
- Ask students to determine how best to calculate various problems without a calculator. If they decide to use mental strategies, have them compute and share their strategies.
- Ask students which questions from a group of computations that they could solve mentally. Explain their thinking and identify the strategy they used.
- Ask the student to generate addition or subtraction number sentences using only decimal numbers that would result in an answer which is close to 50. Share their work.
- Ask the students to use a calculator, the digits 7, 5, 1, and 2, and the symbols "+", "=", and "." (decimal point) to produce 7.8 on the display.
- Have the student model a subtraction problem as subtraction or a missing addend using a variety of models and pictorial representations. Have students record their procedure using symbols.

**Possible Models**: calculators, number lines, place value chart, base ten blocks, hundred grids

SCO: N11: Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:

- using compatible numbers
- estimating sums and differences
- using mental math strategies to solve problems.

[C, ME, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the student to count back (or count on) the change from \$5.00, if the bill totalled \$3.59
- Ask the student to make up a problem with multi-digit numbers for which the calculation could be done mentally. Have them solve it and explain their thinking.
- Ask: How can you know that  $3.65 + 5.35 < 10$  without actually completing the addition? (Observe if the student applied the compatible number strategy).
- Show the student  $\$44.98 + \$3.98 + \$10.99$ . Ask him/her to calculate the sum mentally.
- Ask students to find the difference for  $2.3 - 1.8$  or other similar computations and explain how they got their answer.
- Tell students that to solve  $9.7 - 8.6$ , Syesha thought  $86 + 11$  are 97. Explain her thinking.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.**

[C, CN, PS, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>PR1</b> Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).</p> <p><b>PR2</b> Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).</p>	<p><b>PR1</b> Identify and describe patterns found in tables and charts, including a multiplication chart.</p>	<p><b>PR1</b> Determine the pattern rule to make predictions about subsequent elements.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Mathematics is often referred to as the science of patterns, since patterns are found in every mathematical concept and in everyday contexts. Patterns are also found in physical and geometric situations as well as in numbers. The same pattern can be found in many different forms (Van de Walle & Lovin, vol. 2, 2006, p. 290).

Students should be encouraged to identify and explain patterns that can be found in a variety of tables and charts, including addition and multiplication tables. These patterns can then be used to help students determine an unknown sum or difference. Students should be encouraged to find and explain patterns that occur in tables. It is important that students understand they can use these patterns to determine unknown products or quotients. Students should be familiar with tables that list either all of the multiplication facts or some portion of them. For example, the three times table might be shown as:

×	0	1	2	3	4	5	6	7	8	9
3	0	3	6	9	?	15	18	21	24	27

Students should also explore the many patterns in the hundred chart. The hundred chart is a useful model to provide opportunities for students to find and describe a variety of patterns as well as identifying missing elements and errors. Students should use vocabulary, such as **vertical**, **horizontal**, **diagonal**, **row**, and **column** to help describe patterns.

SCO: PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.

[C, CN, PS, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Describe the pattern found in a given table or chart
- Identify and describe a variety of patterns in a multiplication chart.
- Determine the missing element(s) in a given table or chart.
- Identify error(s) in a given table or chart, such as shown below.

×	0	1	2	3	4	5	6	7	8	9
4	0	1	8	12	16	20	22	28	32	36

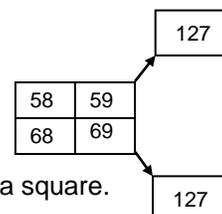
**SCO: PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.**  
[C, CN, PS, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*



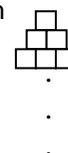
### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Explore the many patterns in the hundred chart. For example, select four numbers that form a square. Add the two numbers on the diagonal, such as,  $59 + 68$  and  $58 + 69$ . The sums are equal.
- Explore patterns found on an addition chart, such as:
  - only even numbers are located on the main diagonal (upper left to lower right), so the sum of a number with itself is always even;
  - the numbers increase by ones across a row, since one more is added for each step right;
  - all of the 8s are on one diagonal line, since each time an addend is one greater, the other must be one less ;
  - there are three 2s, four 3s, five 4s, etc.;
  - the diagonals of any four numbers that form a square will have the same sum.
- Explore patterns found on a multiplication chart, such as:
  - the numbers in each row and column increase by the same amount;
  - the square numbers are found on the left-right diagonal;
  - the numbers on the left-right diagonal increase by 1, 3, 5, 7,....;
  - row 4 is double row 2, row 6 is double row 3;
  - the grid is symmetrical (i.e., numbers are the same both above and under the left-right diagonal);
  - when you add the corresponding products of rows 2 and 3, you get the product in row 5; for example,  $2 \times 4$  (8) plus  $3 \times 4$  (12) is the same as  $5 \times 4$  (20);
  - when you "cross add" any 4 numbers that form a square on the grid, the sum is always the same; for example,  $2 + 13 = 3 + 12$ . When you "cross add" numbers in the corners of larger "squares", you get the same pattern; for example,  $57 + 79 = 59 + 77$ .

### Suggested Activities

- Provide students with a hundred chart, an addition chart, and a multiplication chart. Ask students to describe some of the patterns they observe.
- Have students find the even and the odd numbers on hundred charts, addition charts, and multiplication charts and see if they can find any patterns.
- Have students extend several hundreds charts so they can see from 1 to 100, 101 to 200, up to 999. On these charts, use coloured counters to cover numbers forming a pattern and explore the place value representation of the covered numbers; for example, the pattern 13, 23, 33, 43, ..., depicted as a vertical column of counters, represents increasing the number by 10 each time.
- Have students explore patterns in different versions of hundred charts by changing the order of the numbers, for example, the numbers can spiral in or out or be placed in a different shaped chart, such as a triangle. Provide students with a blank hundred chart so they can create their own version.
- Ask the students to show how one could use the multiplication chart to practice skip counting.



**Possible Models:** hundred chart, addition and multiplication charts (tables), blank grids

SCO: PR1: Identify and describe patterns found in tables and charts, including a multiplication chart.  
[C, CN, PS, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask the students to explain why some column/rows on a multiplication grid have both even and odd numbers.
- Provide a chart or grid with missing numbers and ask students to fill in the missing numbers.
- Provide the students with a multiplication grid. Ask him/her to describe some of the patterns he/she observes.
- Create a chart/grid/table that has not been used in the class as a model and ask students to identify and explain the patterns that can be found on the chart/grid/table.
- Create a chart/grid containing errors and ask students to identify the errors and correct them.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: <b>PR2: Reproduce a pattern shown in a table or chart using concrete materials.</b> [C, CN, V]			
<b>PR3: Represent and describe patterns and relationships using charts and tables to solve problems.</b> [C, CN, PS, R, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

### Scope and Sequence of Outcomes

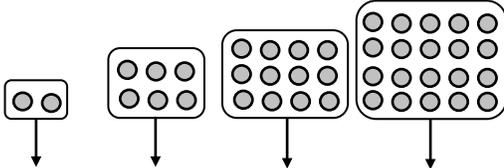
Grade Three	Grade Four	Grade Five
<p><b>PR1</b> Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).</p> <p><b>PR2</b> Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).</p>	<p><b>PR2</b> Reproduce a pattern shown in a table or chart using concrete materials.</p> <p><b>PR3</b> Represent and describe patterns and relationships using charts and tables to solve problems.</p>	<p><b>PR1</b> Determine the pattern rule to make predictions about subsequent elements.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Once a table or chart is developed, students have two representations of a pattern: the one created with the drawing or materials and the numeric version that is in the table or chart. When looking for relationships, some students focus on the table and others will focus on the physical pattern. It is important for students to see that whatever relationships they discover, they exist in both forms. When a relationship is found in a table, challenge students to see how that pattern plays out in a physical version (Van de Walle & Lovin, vol. 2, 2006, p. 295).



Step	1	2	3	4	5	6	?	...	10
No. of dots	2	6	12	20	30	?	?	...	?

Growing patterns also have a numeric component; the number of objects in each step. A table or “T-chart” can be constructed to explore this. Once a table is used for the growing pattern, the materials may become unnecessary. This leads to the next step in pattern exploration which would be to predict what will happen at a particular step (Van de Walle & Lovin, vol. 2, 2006, p. 293-294).

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**

[C, CN, V]

**PR3: Represent and describe patterns and relationships using charts and tables to solve problems.**

[C, CN, PS, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

#### **PR2**

- Use a model to create a concrete representation of a given pattern displayed in a table or chart.
- Explain why the same relationship exists between the pattern in a table and its concrete representation.

#### **PR3**

- Extend patterns found in a table or chart to solve a given problem.
- Translate the information provided in a given problem into a table or chart.
- Identify and extend the patterns in a table or chart to solve a given problem.

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**

[C, CN, V]

**PR3: Represent and describe patterns and relationships using charts and tables to solve problems.**

[C, CN, PS, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Have students not only practice extending patterns with materials and drawings but translate patterns from one medium to another. For example, red and blue pattern blocks become letters or triangles and squares translate to coloured tiles. Have students explain how these patterns are mathematically alike.
- Engage students in constructing growing patterns with different materials (toothpicks, linking cubes, etc.). They may draw growing patterns on grid paper as well. Ask students to describe what is happening as the pattern continues. How is the new step related to the previous one?

### Suggested Activities

- Present students with a geometric design series and have them extend the pattern and develop a "T-chart" to go with it. Ask students what the 10<sup>th</sup> step would be, 12<sup>th</sup>, 20<sup>th</sup>, etc. For example:



- Provide a table or "T-chart" involving one arithmetic operation in the pattern, such as the one below. Describe what the data could be about and complete the table.

1	2	3	4	5	6	7	8	9
4	8	12	16	?	?	?	?	?

**Possible Models:** grids, tables, charts, number lines, linking cubes, toothpicks

SCO: **PR2: Reproduce a pattern shown in a table or chart using concrete materials.**

[C, CN, V]

**PR3: Represent and describe patterns and relationships using charts and tables to solve problems.**

[C, CN, PS, R, V]

### **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

#### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

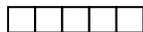
Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

#### **Whole Class/Group/Individual Assessment**

- Provide a table or chart and have students use a model to create a concrete representation of the given pattern displayed in the table or chart.
- Provide several examples of tables and their concrete representations. Ask students to match each table to its concrete representation.
- Ask students to fill in the missing parts of a table or chart. Drawings or materials may be used to help discover the missing parts.
- Ask students to solve the following problem. John was making trains using linking cubes:



First Train



Second Train



Third Train

If he continues to build trains this way, how many blocks will he use in the 7th train?

Ask the students to look for a pattern and create a table to display the information and solve the problem.

Train	Number of cubes
1	1
2	5
3	9
4	
5	
6	
7	

### **FOLLOW-UP ON ASSESSMENT**

#### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

**SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.**  
[CN, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>PR1</b> Demonstrate an understanding of increasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).</p> <p><b>PR2</b> Demonstrate an understanding of decreasing patterns by: describing; extending; comparing; creating patterns using manipulatives, diagrams, sounds and actions (numbers to 1000).</p>	<p><b>PR4</b> Identify and explain mathematical relationships using charts and diagrams to solve problems.</p>	<p><b>PR1</b> Determine the pattern rule to make predictions about subsequent elements.</p>

### ELABORATION

#### Guiding Questions:

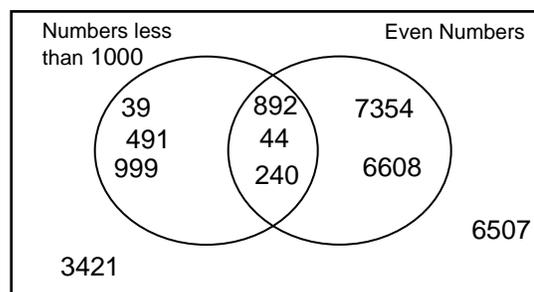
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

In everyday life things are sorted by comparison relationships, for example by colour and size. Such relationships also apply to number, as numbers also have attributes or certain characteristics which make them similar or different from other numbers. Students need to explore this particular concept of numbers by being involved in experiences where they are expected to recognize, describe and identify relationships and number characteristics. Sorting and classifying objects and numbers will help students with organizing and categorizing data. **Sorting** is the action of grouping (or organizing) objects (or data); **classification** (or categorization) is the naming of the groups of objects (or data).

By grade 4, students are expected to use more sophisticated sorting tools such as a **Carroll** or **Venn diagram**. These organizational tools are particularly useful as a form of data display when the categories for the sorting situation overlap. A Venn diagram is typically drawn with one, two or three circles. It is important to draw a rectangle around Venn diagrams to represent the “universe” or entire group that is being sorted. This will show the items that do not fit the attributes of the circle(s) outside of them, but within the rectangle. Carroll diagrams are tables that work much like Venn diagrams. For Carroll diagrams, two attributes are being used for sorting, with one attribute of each characteristic being the focus (Small, 2008, p. 521). A table is created with four cells to show the four possible combinations of these two attributes. Either the items themselves, or the count of how many items of each type, are put in the cells.

	Even	Odd
Numbers less than 1000	892, 44, 240	39, 491, 999
Numbers more than 1000	7354, 6608	3421, 6507

Carroll Diagram



Venn Diagram

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.  
[CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Complete a Carroll diagram by entering given data into correct squares to solve a given problem.
- Determine where new elements belong in a given Carroll diagram.
- Solve a given problem using a Carroll diagram.
- Identify a sorting rule for a given Venn diagram.
- Describe the relationship shown in a given Venn diagram:
  - when the circles intersect;
  - when one circle is contained in the other;
  - when the circles are separate.
- Determine where new elements belong in a given Venn diagram.
- Solve a given problem by using a chart or diagram to identify mathematical relationships.

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.  
[CN, PS, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Introduce the notion of how cross-classification is shown in Venn diagrams by setting out two hoops, side by side, and label each one with a sorting rule. Ensure the sorting rules and objects to be sorted lend themselves to cross-classification. Note that the circles of a Venn diagram do not have to overlap. They can be two separate circles if the attributes are exclusive. They could also be separate if the items involved in the sort do not exhibit the same characteristics, even if they have the potential to do so (Small, 2008, p. 521). If only one attribute is being used to sort the objects, then there would only be one circle in the Venn diagram.
- Reinforce the proper math vocabulary during sorting activities. The word “and” indicates that each item in the group would have all attributes of both categories where “or” makes the distinction between the two categories under consideration.
- Ensure that students include all of the data being considered from their sorting situation in their Venn or Carroll diagram. The rectangle drawn around the Venn diagram is used to show that the all of the data, including items that did not match the sorting criteria, has been considered.
- Post a list of possible attributes (or characteristics) of numbers and encourage students to refer to the list as they examine Venn or Carroll diagrams involving numbers.

### **Suggested Activities**

- Give students various 3-D objects or 2-D shapes. One student selects six objects or shapes, chooses 2 mystery attributes and then sorts them. The other(s) then attempt to guess the sorting rule.
- Have students create a set of ten 3- or 4-digit numbers and sort them using two attributes. Request that they write the sorting rule.
- Provide students with data to organize using a both Venn and Carroll diagram. Have students reflect on which is their preferred sorting organizational tool and justify their choice.
- Have students sort a set of numbers in different ways and explain their sorting rule(s).

**Possible Models:** attribute blocks, 2-D shapes, 3-D objects, money, collection of various objects to sort, cards or tiles (commercial or teacher-made)

SCO: PR4: Identify and explain mathematical relationships using charts and diagrams to solve problems.  
[CN, PS, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Prepare various pre-sorted 2-D shapes or 3-D objects in Venn diagrams. Hold up different additional objects and ask students where it should go in the Venn diagram.
- Provide an unlabelled Venn diagram, containing pre-sorted sets of numbers, and ask students to determine the sorting rule and add one more number to each subset.
- Give students various numeral cards containing numbers up to 4-digits and have students create a labelled Venn or Carroll diagram.
- Provide a completed Carroll diagram and present students with additional numbers which might have one attribute, both attributes, or neither attribute. Have them explain where they should be placed in the diagram.
- Have students compare a completed Venn diagram to a related Carroll diagram and ask them to determine if they show the same information. Have them explain their thinking.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: <b>PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.</b> [CN, PS, R]			
PR6: <b>Solve one-step equations involving a symbol to represent an unknown number.</b> [C, CN, PS, R, V]			
[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<b>PR3</b> Solve one-step addition and subtraction equations involving a symbol to represent an unknown number.	<b>PR5</b> Express a given problem as an equation in which a symbol is used to represent an unknown number. <b>PR6</b> Solve one-step equations involving a symbol to represent an unknown number.	<b>PR2</b> Express a given problem as an equation in which a letter variable is used to represent an unknown number (limited to whole numbers). <b>PR3</b> Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions.

### ELABORATION

#### Guiding Questions:

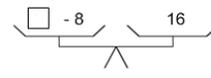
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

The various representations of patterns, including unknowns, provide valuable tools in making generalizations of mathematical relationships.

**Equality** is used to express **relationships**. The symbols used on either side of the equal sign represent a quantity. The equal sign is "a symbol of equivalence and balance" (NCTM, 2000, p. 39).

Students should be given opportunities to explore equivalence using models and pictures before they begin to represent the equations symbolically. An **equation** is a mathematical statement that includes an equal sign and may have been called a **number sentence** in earlier grades. Students should be comfortable using various symbols to represent the **unknown number** in an equation, for example, a square, circle, triangle, or other shapes.

Display a number of samples of balance scales, such as those shown below. Have students write an equation for each balance scale and then solve it. For example, for the first model, students would write the equation,  $8 + \square = 20$ , and the solution would be:  $\square = 12$ . Include examples of scales with the single number on the left side of the scale, so students have opportunities to write equations in different structures (e.g.,  $36 = \square \times 9$ ).



SCO: PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.

[CN, PS, R]

PR6: Solve one-step equations involving a symbol to represent an unknown number.

[C, CN, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

### **PR5**

- Explain the purpose of the symbol, such as a triangle or circle, in a given addition, subtraction, multiplication or division equation with one unknown, e.g.,  $36 \div \square = 6$ .
- Express a given pictorial or concrete representation of an equation in symbolic form.
- Identify the unknown in a story problem, represent the problem with an equation and solve the problem concretely, pictorially or symbolically.
- Create a problem in context for a given equation with one unknown.

### **PR6**

- Solve a given one-step equation using models.
- Solve a given one-step equation using guess and test.
- Describe, orally, the meaning of a given one-step equation with one unknown.
- Solve a given equation when the unknown is on the left or right side of the equation.
- Represent and solve a given addition or subtraction problem involving a “part-part-whole” or comparison context (situation) using a symbol to represent the unknown.
- Represent and solve a given multiplication or division problem involving equal grouping or partitioning (equal sharing) using symbols to represent the unknown.

SCO: PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.

[CN, PS, R]

PR6: Solve one-step equations involving a symbol to represent an unknown number.

[C, CN, PS, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should I use?
- How will I meet the diverse learning needs of my students?

### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Build on the students' knowledge from the previous grade in using equations to write addition, subtraction, multiplication and division equations. Connect the concrete, pictorial and symbolic representations as the students develop and demonstrate understanding of equations.
- Use everyday contexts for problems to which the students can relate so that they can translate the meaning of the problem into an appropriate equation using a symbol to represent the unknown.
- Review the relationship between addition and subtraction equations as well as the relationship between multiplication and division equations.
- Have the students create problems for a variety of equations using the 4 operations.
- Encourage the students to write equations in various ways to represent the meaning of a given problem. For example,  $14 + \Delta = 37$  or  $\Delta + 14 = 37$ ;  $5 \times \square = 30$  or  $\square \times 5 = 30$ . Note that the order (commutative) property does not apply to subtraction and division.
- Explain that if the same variable, or unknown number, is used repeatedly in the same equation, then there is only one possible solution for that variable or unknown; e.g., for  $\square + \square = 20$ ; the unique solution is to place 10 in each of the squares. If, however, two different symbols are used, there may be a number of possible solutions, e.g.,  $\square + \Delta = 16$ , some solutions include  $0 + 16$ ,  $7 + 9$ ,  $12 + 4$ .
- Explore interactive websites that have equation balancing activities, such as NCTM's Illumination Activity site( <http://illuminations.nctm.org/ActivityDetail.aspx?ID=26>).

### Suggested Activities:

- Ask students to create problems to represent the following equations:
 

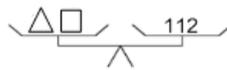
$15 + \square = 24$	$\Delta + 15 = 24$	$24 = 15 + \circ$	$24 = \diamond + 15$
$24 - \square = 15$	$24 - 15 = \nabla$	$15 = 24 - \square$	$\triangleright = 24 - 15$
- Show students a balance scale and ask them to work with a partner to find an equation that is represented for each of the examples below. Possible solutions are shown under the balances.



$$3 \times \square = 24$$

or

$$\square + \square + \square = 24$$



$$\Delta + \square = 112$$



$$2 \times \Delta = 98$$

or

$$\Delta + \Delta = 98$$

**Possible Models:** balance scales, linking cubes

SCO: PR5: Express a given problem as an equation in which a symbol is used to represent an unknown number.

[CN, PS, R]

PR6: Solve one-step equations involving a symbol to represent an unknown number.

[C, CN, PS, R, V]

### **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

#### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

#### **Whole Class/Group/Individual Assessment**

- Tell what the box represents in the following equation:  $15 - \square = 8$ .
- Tell the student that you have 24 marbles and your friend gives you some more marbles. Now you have 32 marbles in all. How many marbles did your friend give you?
  - a. Write an equation to show what is happening in this problem.
  - b. Solve the problem. Explain your thinking.
- Solve the following equation and explain the process.  
 $34 + 5 = \square + 12$
- Solve the following equation and explain your thinking.  
 $\triangle - 13 = 20$
- Tell the student that Lori said that the box in the following equation represents more than one number. Is Lori correct? Why or why not?  
 $6 + 8 = \square + 4$
- Ask the student to explain how to find the missing number in  $4 \times \triangle = 100$ .

### **FOLLOW-UP ON ASSESSMENT**

#### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.**  
[C, CN, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>SS1</b> Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).</p> <p><b>SS2</b> Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.</p>	<p><b>SS1</b> Read and record time using digital and analog clocks, including 24-hour clocks.</p>	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Although students have not had any explicit teaching related to reading and recording time using clocks by grade four, they have had opportunities in previous grades to explore the passage of time and have learned that there are 60 seconds in a minute and 60 minutes in an hour. In grade four, students learn that there are twenty-four hours in a day. Prior to grade four, students will have had many opportunities to learn about the passage of time through their own experiences.

By the end of grade four, students should be able to read and record time on 12-hour and 24-hour **analog** and **digital** clocks. Students should read times on clocks to provide information about relevant situations, focusing on times when special events are going to happen. It is important for students to be able to precisely describe time, telling how many “minutes to” and how many “minutes after” the hour.

Students may want to investigate the meaning for some terminology such as, A.M. and P.M (note: it is also acceptable to write these as a.m. and p.m.). The abbreviation, A.M., is the short form for *ante meridiem* meaning being before noon and P.M. is the abbreviation for *post meridiem* meaning being after noon.

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.**  
[C, CN, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- State the number of hours in a day.
- Express the time orally and numerically from a 12-hour analog clock.
- Express the time orally and numerically from a 24-hour analog clock.
- Express the time orally and numerically from a 12-hour digital clock.
- Describe time orally and numerically from a 24-hour digital clock.
- Describe time orally as “minutes to” or “minutes after” the hour on both a 12-hour and 24-hour clock.
- Explain the meaning of A.M. (a.m.) and P.M. (p.m.), and provide an example of an activity that occurs during the A.M. and another that occurs during the P.M.

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.**  
[C, CN, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Have students explore an analog clock and share their findings. Students will be aware that:
  - the minute hand is at 6 for the “:30” on a digital clock and at 12 or 24 for the “:00”;
  - the hour hand moves during the course of an hour; at the :30, it is halfway between two numbers;
  - the hour and minute hands are different lengths.
- Use an analog clock to introduce the terms “half past”, “quarter after”, and “quarter to” as well as how many “minutes to” and how many “minutes after”.
- Have students read time to the nearest five minutes. It is important that students are comfortable with skip counting by 5. This provides the opportunity for students to relate the numbers on a clock to the five times table.
- Use a clock that shows not only the numbers from 1 to 12, but also the minute amounts from 5 to 55 beside the numbers from 1 to 11. Students should be aware that there are 5 minutes between the numbers on the clock. The short hand on the 3 represents 15 minutes, so two one-minute spaces past the 3 is 17 minutes, etc.

### **Suggested Activities**

- Present the student with a time shown on an analog clock that just has the hour hand. Ask him/her to predict what the time might be. For example, if the hour hand is somewhere between the 4 and the 5, the time could be anything from *five past four* to *five to five* depending on the exact placement of the hour hand. Students could also be asked to name an event/activity that often happens at about that time of day.
- Introduce the terms of analog: a.m. and p.m. Discuss the difference between the two terms and brainstorm activities that would take place during each.
- Ask the student to show, on an analog clock, the time (to the nearest half hour) at which they arrive at school, have lunch, go to bed, etc.
- Discuss when a 24-hour clock would be more appropriate to use than a 12-hour clock.
- Have students track events throughout a specific day by means of a time line divided into 15-minute segments. Students should record the time of the activity or event and note it at the appropriate spot on a time line.
- Have the student make a list of the times when the minute hand and the hour hand just about line up as well as other patterns, such as all of the times that include a 4 in a 24-hour period.
- Have students work in pairs to set up a schedule in which every student will get 10 minutes on the computer, starting at 8:30 a.m. Ask students if everyone in the class can have time on the computer before noon and, if not, how long it will take to finish after lunch. At what time will the last one finish? (Remind students to leave time for recess.)
- **Possible Models:** 12-hour analog clock, 24-analog clock, digital clock that reads 12-hours and 24-hours

SCO: **SS1: Read and record time using digital and analog clocks, including 24-hour clocks.**  
[C, CN, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Ask students what time might it be if the minute hand and hour hand are opposite one another.
- Ask the student to move the hands of an analog clock to match the time shown on a digital clock.
- Ask students to express the time orally and numerically that has been created on a 12-hour analog clock, 24-hour analog clock, and 12-hour digital clock.
- Ask students to name an activity they would typically do in the P.M.? A.M.?
- Ask students how many hours are in a day and a half.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS2: Read and record calendar dates in a variety of formats.**  
[C, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>SS1</b> Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years).</p> <p><b>SS2</b> Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.</p>	<p><b>SS2</b> Read and record calendar dates in a variety of formats.</p>	

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

By grade four, students should already know the days of the week, the months of the year, and the four seasons. As well, students will have already developed a sense of the arrangement of our year in relation to the months and seasons; for example, January is the first month of a new year and is early in our winter season.

Using **calendars** throughout the school year strengthens the students' sense of time. Each month brings a new calendar to explore. Students would be familiar with calendars through their home and school experiences by grade 4. In previous grades, teachers may have explored calendars during explorations of units of time, such as days, weeks, months, and years. Calendars may also have been used to assist in developing number sense and for exploring patterns.

Students need to become aware of the variety of ways **dates** can be recorded. In grade four, students are expected to read, record, and interpret calendar dates in a variety of ways, including words and numbers. It is important for students to be familiar with different formats for dates since there are several that are acceptable and will be encountered in their daily lives. The International Organization for Standardization (ISO) has identified a standard notation that many countries, including Canada, have adopted. It always starts with the year, then the month, and the last digits are the day (yyyy-mm-dd). It always uses four digits for the year and all other numbers less than 10 are recorded with a leading zero (e.g., 2009-01-04 is how January 4, 2009 would be recorded).

SCO: **SS2: Read and record calendar dates in a variety of formats.**  
[C, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Write dates in a variety of formats, e.g., *yyyy/mm/dd*, *dd/mm/yyyy*, March 21, 2006, *dd/mm/yy*.
- Relate dates written in the format *yyyy/mm/dd* to dates on a calendar.
- Identify possible interpretations of a given date, e.g., 06/03/04.

SCO: **SS2: Read and record calendar dates in a variety of formats.**  
[C, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide groups of students with yearly calendars. Have students explore the calendars and share their findings. Students should focus on the different formats of how dates are written.
- Send students on a scavenger hunt and have them bring in different dates from magazines, posters, items printed from the Internet, cheques and newspapers. Share, discuss and display the variety of formats as a class.
- Have students predict how many days and/or weeks there are in a year. Verify using calendars.
- Have them explore what calendar dates can be confused with other dates when they are interpreted using various formats.
- Investigate a special holiday which has a date that fluctuates, such as Labour Day. Have students record the date(s) of this holiday over the past five years in different formats. Share their findings.

### **Suggested Activities**

- Ask students to write about their favourite format for recording a calendar date and justify their choice.
- Have students interpret a particular date such as 06/04/03. Discuss that there is no standard or consistent format and why some dates may be misinterpreted unless you know the format.
- Provide students with a list of dates recorded in the ISO standard notation (yyyy-mm-dd) and have them order them from past to present.

**Possible Models:** calendars, electronic devices that display dates

SCO: **SS2: Read and record calendar dates in a variety of formats.**  
[C, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Show the student a calendar for the year. Ask him/her to point out the day's date. Have them record it using the format month/day/year.
- Ask the student to identify two calendar dates which cannot be confused with other dates when they are interpreted regardless of the format.
- Have the student write their birth date using 3 different formats.
- Have the student identify their favourite day of year and write the date in ISO standard notation (yyyy-mm-dd).

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- recognizing that area is measured in square units
- selecting and justifying referents for the units  $\text{cm}^2$  or  $\text{m}^2$
- estimating area by using referents for  $\text{cm}^2$  or  $\text{m}^2$
- determining and recording area ( $\text{cm}^2$  or  $\text{m}^2$ )
- constructing different rectangles for a given area ( $\text{cm}^2$  or  $\text{m}^2$ ) in order to demonstrate that many different rectangles may have the same area.

[C, CN, ME, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>SS5</b> Demonstrate an understanding of perimeter of regular and irregular shapes by: estimating perimeter, using referents for cm or m; measuring and recording perimeter (cm, m); constructing different shapes for a given perimeter (cm, m); to demonstrate that many shapes are possible for a perimeter.</p>	<p><b>SS3</b> Demonstrate an understanding of area of regular and irregular 2-D shapes by:</p> <ul style="list-style-type: none"> <li>• recognizing that area is measured in square units</li> <li>• selecting and justifying referents for the units <math>\text{cm}^2</math> or <math>\text{m}^2</math></li> <li>• estimating area by using referents for <math>\text{cm}^2</math> or <math>\text{m}^2</math></li> <li>• determining and recording area</li> <li>• constructing different rectangles for a given area (<math>\text{cm}^2</math> or <math>\text{m}^2</math>) in order to demonstrate that many different rectangles may have the same area.</li> </ul>	<p><b>SS1</b> Design and construct different rectangles, given either perimeter or area, or both (whole numbers), and make generalizations.</p>

### ELABORATION

#### Guiding Questions:

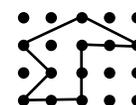
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should understand that the area of a shape can be expressed as the number of units required to cover a certain surface. Van de Walle and Lovin define area as "a measure of the space inside a region or how much it takes to cover a region" (Van de Walle & Lovin, vol. 2, 2006, p. 234). The **square unit** is the most efficient unit to use for measuring area.

It is helpful for students to use a **referent** for the single unit of measure and iterate this unit mentally to obtain the estimate (e.g., use the size of the fingernail on your finger or thumb as a referent for  $1 \text{ cm}^2$ ). Once students have developed the meaning of measurement, it is time to move on to connect multiplication in an **array** format to determine the area of rectangles (Van de Walle & Lovin, vol. 2, 2006, p.263). Students should relate the area of a rectangle to the product of the numbers describing its length and width. Conversely, any factor of the number representing the area of a rectangle can be one dimension of a rectangle with that area. For example, consider rectangles with an area of 8 square units. It is important for students to explore not only the areas of rectangles, but areas of other shapes as well. Through these investigations students should recognize that objects of different shapes can have the same area. Encourage students to find shapes using partial squares.



Opportunities should be provided for students to estimate and calculate the area of various surfaces. Laying an acetate centimetre grid over objects is helpful when determining surface area. Students might investigate the area of shapes drawn on centimetre dot paper. Strategies for doing this include adding squares and half squares within the figure; placing a rectangle around the shape, determining its area, and subtracting the area of the "extra" pieces.



SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- recognizing that area is measured in square units
- selecting and justifying referents for the units  $\text{cm}^2$  or  $\text{m}^2$
- estimating area by using referents for  $\text{cm}^2$  or  $\text{m}^2$
- determining and recording area ( $\text{cm}^2$  or  $\text{m}^2$ )
- constructing different rectangles for a given area ( $\text{cm}^2$  or  $\text{m}^2$ ) in order to demonstrate that many different rectangles may have the same area.

[C, CN, ME, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Describe area as the measure of surface recorded in square units.
- Identify and explain why the square is the most efficient unit for measuring area.
- Provide a referent for a square centimetre and explain the choice.
- Provide a referent for a square metre and explain the choice.
- Determine which standard square unit is represented by a given referent.
- Estimate the area of a given 2-D shape using personal referents.
- Determine the area of a regular 2-D shape and explain the strategy.
- Determine the area of an irregular 2-D shape and explain the strategy.
- Construct a rectangle for a given area.
- Demonstrate that many rectangles are possible for a given area by drawing at least two different rectangles for the same given area.

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- recognizing that area is measured in square units
- selecting and justifying referents for the units  $\text{cm}^2$  or  $\text{m}^2$
- estimating area by using referents for  $\text{cm}^2$  or  $\text{m}^2$
- determining and recording area ( $\text{cm}^2$  or  $\text{m}^2$ )
- constructing different rectangles for a given area ( $\text{cm}^2$  or  $\text{m}^2$ ) in order to demonstrate that many different rectangles may have the same area.

[C, CN, ME, PS, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

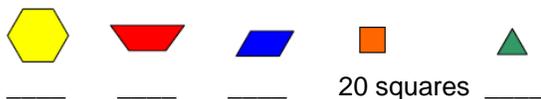
- Use referents for area and estimating area. Review that referents are familiar objects to which students can refer for estimating (e.g., the width of the “little” finger is about 1 cm). Ask the students to suggest a suitable referent for  $1 \text{ cm}^2$  and explain why they think it would work. Have them use this referent to estimate the area of a book cover. Ask them to check their estimate by finding the area of the book cover. Discuss possible referents for  $1 \text{ m}^2$ . Have the students use their referents to estimate the area of a large tabletop or a section of the classroom floor and check their estimates.
- Have students use colour tiles or grid paper to investigate the numbers from 1 to 30 to see how many different rectangles can be made for each. Students should record their results and look for patterns.
- Use a transparency of a centimetre grid to confirm the estimate of an area of an irregular shape.

### Suggested Activities

- Have students explore how a diagonal of rectangle(s) divides the shape in half.



- Provide the students with rectangular papers that each measure 10 cm by 13 cm. Have them estimate how many copies of each shape of pattern block it would take to cover the rectangle. Then have the students measure the area using each of the shapes in turn.



- Make the design to the right on an overhead geoboard or interactive whiteboard and ask students to explain various ways to find the area. Have the students create the same shape on their geoboard and alter the shape to increase the area by  $1 \text{ cm}^2$ .



- Provide the students with tiles and centimetre grid paper. Give them the following instructions: For each of the areas from  $1 \text{ cm}^2$  to  $20 \text{ cm}^2$ , find all the possible rectangular arrays using whole numbers. For example, the possible arrays for an area of  $6 \text{ cm}^2$  would be as follows:

1 row of 6 square units 2 rows of 3 square units.



**Possible Models:** pattern blocks, centimetre grid paper, geoboards, colour tiles, transparency grid paper

SCO: **SS3: Demonstrate an understanding of area of regular and irregular 2-D shapes by:**

- recognizing that area is measured in square units
- selecting and justifying referents for the units  $\text{cm}^2$  or  $\text{m}^2$
- estimating area by using referents for  $\text{cm}^2$  or  $\text{m}^2$
- determining and recording area ( $\text{cm}^2$  or  $\text{m}^2$ )
- constructing different rectangles for a given area ( $\text{cm}^2$  or  $\text{m}^2$ ) in order to demonstrate that many different rectangles may have the same area.

[C, CN, ME, PS, R, V]

## ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

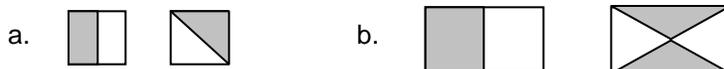
### Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### Whole Class/Group/Individual Assessment

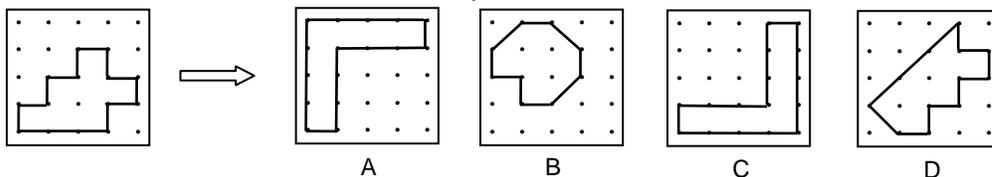
- Ask students to predict how many different arrays can be made to represent  $36 \text{ cm}^2$ . Draw all of the arrays to check your prediction.
- Have students estimate the area for each of the following pairs of congruent shapes. Decide if the shaded part has the same area in each pair of shapes. Explain your thinking.



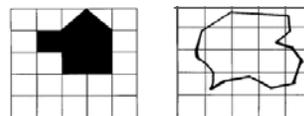
- Ask the student to estimate the area of a rectangle and explain what referent he or she used.
- Explain why area is measured in square units.
- Find the area of the shaded part.  
The area of the entire design to the right is  $12 \text{ m}^2$ .  
Explain your thinking.



- Ask students to circle the letters of the shapes that have the same area as the first one on the left.



- Ask why it is easier to find the area of the shape on the left than the one on the right.  
Give an estimate for the shape on the right.  
Explain your thinking.



## FOLLOW-UP ON ASSESSMENT

### Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SS4: Describe and construct rectangular and triangular prisms.**  
[C, CN, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<b>SS6</b> Describe 3-D objects according to the shape of the faces, and the number of edges and vertices.	<b>SS4</b> Describe and construct rectangular and triangular prisms.	<b>SS5</b> Describe and provide examples of edges and faces of 3-D objects and sides of 2-D shapes that are: parallel; intersecting; perpendicular; vertical; horizontal.

### ELABORATION

#### Guiding Questions:

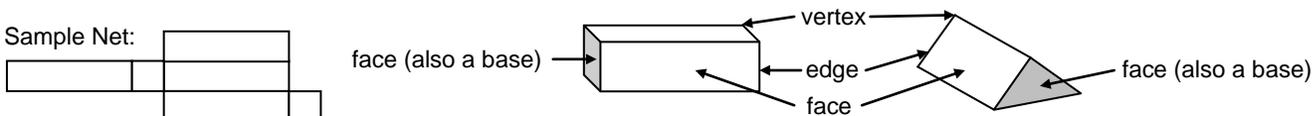
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Students should draw upon their previous knowledge of two dimensional polygons to assist them in their identification and description of prisms. In the earlier grades students classified geometric shapes by general characteristics and will now develop more detailed ways to describe objects. Students will identify properties of shapes and objects, and learn to use proper mathematical vocabulary to describe them.

All prisms have **faces**, two of which are customarily referred to as **bases**. These two bases may take the shape of any polygon. For clarification purposes, prisms can be thought of as having two names. The “first name” refers to the shape of the bases and a second name, which is **prism** (e.g., triangular prism, rectangular prism). Some students may be keen to identify other prisms such as hexagonal prisms or square prisms (square prisms fall into the category of rectangular prisms because a square is a rectangle). In grade four, instruction is focused on **rectangular prisms** and **triangular prisms**. Sets of 3-D objects usually include a variety of prisms. Students also need to be able to identify examples of rectangular and triangular prisms in their environment.

A good way to explore shapes is to use smaller shapes or tiles to create larger shapes. Pattern blocks are very good for this, but many other materials can be used. Although the pattern block pieces have been primarily used to represent 2-D shapes, they are prisms. Stacking a number of triangle or square pattern blocks would provide examples of different prisms. This stacking would help students conceptualize the uniform nature of prisms. Students can also make skeletal models for prisms, using rolled newspapers and tape, straws and string, or toothpicks and miniature marshmallows.

Students should be given copies of **nets** of rectangular and triangular prisms to cut out and fold to construct the prisms. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. Students should recall from grade three that these shapes are the **faces** of the 3-D object and are one of the key **attributes** students should use to identify rectangular and triangular prisms. Other attributes students should consider when identifying 3-D objects are the number of **edges** and **vertices** (from grade three) and **congruency** (see SS6). In addition to cutting out and assembling prepared nets, it is also expected that students will draw their own nets for rectangular and triangular prisms and explore other possible nets for these prisms. Have students visualize the folding and unfolding of the nets and then use materials to explore whether the net will successfully construct the prism.



SCO: **SS4: Describe and construct rectangular and triangular prisms.**  
[C, CN, R, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify and name common attributes of rectangular prisms from given sets of rectangular prisms.
- Identify and name common attributes of triangular prisms from given sets of triangular prisms.
- Sort a given set of rectangular and triangular prisms using the shape of the base.
- Construct and describe a model of rectangular and triangular prisms using materials, such as pattern blocks or modeling clay.
- Construct rectangular prisms from their nets.
- Construct triangular prisms from their nets.
- Identify examples of rectangular and triangular prisms found in the environment.

SCO: **SS4: Describe and construct rectangular and triangular prisms.**  
[C, CN, R, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Provide groups of students with a variety of rectangular and triangular prisms. Have students explore the attributes of these objects and share their findings.
- Encourage students to use the attributes of any prism (number of faces, number of edges, number of vertices, or shapes of the faces) to describe prisms.
- Determine if students recognize the same prism can be built by piling vertically or horizontally when building objects from the base, (i.e., orientation).
- Work with nets to investigate attributes, including alignment of faces, to determine if the net could successfully construct the 3-D object.

### **Suggested Activities**

- Stack pattern blocks to make rectangular prisms and triangular prisms. Describe how they are alike and how they are different.
- Provide students with various nets of prisms for them to construct. Have students label each face of their model using the words “face” and “base” as well as identify their 3-D object.
- Have the students trace on paper the various faces of the different prisms to make its net. Have the students cut out the net and fold it up around the shape to check if it works. Ask students record this net on grid paper. Have the student then cut off one of the faces and investigate the possible places it could be reattached to make a new successful net. Record each new net on grid paper.
- Provide students with a square or rectangular prism and an 11-pin x 11-pin geoboard. Ask students to use elastics to construct a net for the prism and discuss how they might move one of the faces to make a new net for the same prism. Have them check by recording the new net on square dot paper and cutting it out.
- Provide the students with one of the 12 pentomino pieces (2-D shapes made by joining 5 squares along full sides). Ask if it could “fold” to make a box with no top. Have students trace this pentomino piece and then add a square for the top of the box. Ask: In how many places can this square be added? (Note: This can be cut from grid paper.) Example of a pentomino that could make a box: 
- Tell the students that this diagram is part of a net for a square (rectangular) prism. Ask them to complete the net by drawing the three additional faces that are needed. 

**Possible Models:** pattern blocks, toothpicks/straws, modeling clay, isometric paper, geoboards, pentominoes, grid paper, linking cubes, Polydrons®

SCO: **SS4: Describe and construct rectangular and triangular prisms.**  
[C, CN, R, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/Individual Assessment**

- Have students name the prism that best represents various real-life examples of 3-D objects (e.g., a book would be represented by a rectangular prism).
- Ask students to build skeletal models of two different triangular prisms. Ask students how they are the same and different.
- Have student work together to sort a collection of 3-D objects into two groups: rectangular and triangular prisms. Ask: what are the attributes of the shapes that made them alike? How are they different? What makes a rectangular prism a cube? What kind of prism would you have if you built from a rectangular base?
- Listen as students construct nets on geoboards and/or using grid paper. Note whether students are discussing the attributes of the object and using the correct vocabulary (i.e., faces, edges, vertices).
- Ask the students to add additional faces to complete partial nets that will successfully construct a rectangular or triangular prism.
- Give small groups of students a set of 4 or 5 nets of rectangular or triangular prisms. Each set should consist of one net that can be made into the 3-D object, and 3 or 4 others which will not construct the 3-D object. Have students analyse the nets, without manipulating them, to determine which one of the nets in the group could be used to create the 3-D object. Have students justify their selection, and then test their prediction.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: <b>SS5: Demonstrate an understanding of line symmetry by:</b> <ul style="list-style-type: none"> <li>• identifying symmetrical 2-D shapes</li> <li>• creating symmetrical 2-D shapes</li> <li>• drawing one or more lines of symmetry in a 2-D shape.</li> </ul> [C, CN, V]			
<b>SS6: Demonstrate an understanding of congruency, concretely and pictorially.</b> [C, CN, V]			
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
	<b>SS5</b> Demonstrate an understanding of line symmetry by: <ul style="list-style-type: none"> <li>• identifying symmetrical 2-D shapes</li> <li>• creating symmetrical 2-D shapes</li> <li>• drawing one or more lines of symmetry in a 2-D shape.</li> </ul> <b>SS6</b> Demonstrate an understanding of congruency, concretely and pictorially.	<b>SS8</b> Identify a single transformation including a translation, a rotation and a reflection of 2-D shapes.

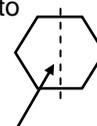
### ELABORATION

#### Guiding Questions:

- What do I want my students to learn?
- What do I want my students to understand and be able to do?

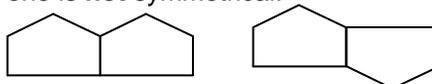
**Congruency** and **symmetry** are **geometric properties**. These properties can be used to determine what makes shapes alike and different. Two 2-D shapes are congruent if they are **identical in shape and size**. Students sometimes do not understand the difference between the math term *congruent* and the everyday term *the same*. It is important to recognize that the term *congruent* applies only to size and shape; not colour or orientation. Symmetrical 2-D shapes are geometric figures "that can be folded in half so that the two parts are congruent" (Alberta Education, 1990, p. 205).

Symmetrical and congruent shapes are closely connected. Any symmetrical shape can be divided into two congruent parts along the line of symmetry; however, not every **composite shape** made up of congruent figures is symmetrical. For example, this regular hexagon is symmetrical. The **line of symmetry** shown in the diagram divides the hexagon into two congruent shapes (pentagons).



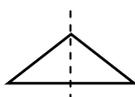
Line of symmetry

The two composite shapes below are constructed using two congruent pentagons. The first composite shape **is** symmetrical, but the second one is **not** symmetrical.



Students should begin to appreciate that reflective symmetry is a characteristic of some polygons and not others. These polygons can be described by stating how many lines of reflective symmetry they have. A shape remains the same size and shape when transformed using translations, reflections or rotations (i.e., the object and the image in these transformations are congruent). Symmetrical shapes form a subset of reflections. A reflection results in a symmetrical composite shape when the mirror line used to reflect a shape aligns with one side of the shape as shown below.

This reflection results in a composite shape that is symmetrical. The mirror line and the line of symmetry coincide.



This reflection does not result in a composite shape, but rather two separate shapes that are congruent. The mirror line is shown, but it is not the line of symmetry.



SCO: **SS5: Demonstrate an understanding of line symmetry by:**

- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape.

[C, CN, V]

**SS6: Demonstrate an understanding of congruency, concretely and pictorially.**

[C, CN, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

### **SS5**

- Identify the characteristics of given symmetrical and non-symmetrical 2-D shapes.
- Sort a given set of 2-D shapes as symmetrical and non-symmetrical and explain the process.
- Complete a symmetrical 2-D shape given half the shape and its line of symmetry and explain the process.
- Identify lines of symmetry of a given set of 2-D shapes and explain why each shape is symmetrical.
- Determine whether or not a given 2-D shape is symmetrical by using a Mira or by folding and superimposing.
- Create a symmetrical shape with and without manipulatives and explain the process.
- Provide examples of symmetrical shapes found in the environment and identify the line(s) of symmetry.
- Sort a given set of 2-D shapes as those that have no lines of symmetry, one line of symmetry or more than one line of symmetry.

### **SS6**

- Determine if two given 2-D shapes are congruent and explain the strategy used.
- Create a shape that is congruent to a given 2-D shape and explain why the two shapes are congruent.
- Identify congruent 2-D shapes from a given set of shapes shown in different orientations.
- Identify corresponding vertices and sides of two given congruent shapes.
- Explain the connections between congruence and symmetry using 2-D shapes.
- Find examples of congruent 2-D shapes in the environment.

SCO: **SS5: Demonstrate an understanding of line symmetry by:**

- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape.

[C, CN, V]

**SS6: Demonstrate an understanding of congruency, concretely and pictorially.**

[C, CN, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

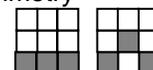
### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Use everyday contexts to introduce congruence and symmetry, drawing on the students' prior experiences in the real world and knowledge of 2-D shapes.
- Include many hands-on activities to establish the concept of congruence prior to symmetry.
- Ask students to create a square using geoboards or linking cubes. Say: Some of these squares are congruent and some are not. Give clues such as Greg's square is not congruent to Sarah's square, but it is congruent to Mei's. Continue giving clues until students discover what congruence means.
- Explore that the line of symmetry can be vertical, horizontal, or diagonal.
- Have students explore that reflective symmetry is a characteristic of some polygons, but not all. Polygons can be described by stating the number of lines of reflective symmetry they have. For example, students should discover that a square has 4 lines of reflective symmetry.
- Provide students with experiences to help them understand that a symmetry line is where a polygon can be folded onto itself so that each half matches exactly, or where a mirror can be placed so that the reflection on one side matches the shape on the other.

### **Suggested Activities**

- Have students create congruent designs on geoboards and draw the designs on square dot paper or grid paper. Students could cut out one design from the dot paper and superimpose it on the other design to check for congruency. It is important to test for congruency because shapes in different orientations may not appear to be congruent even when they are.
- Provide a variety of shapes and ask the students to sort them, grouping those with reflective symmetry and those without reflective symmetry.
- Ask each student to draw a picture of a shape or create a design that exhibits symmetry
- Ask students to draw examples of triangles with symmetry and triangles without symmetry.
- Have students draw on squared dot paper examples of the different quadrilaterals. Cut them out and fold them to find the lines of symmetry. Use pictures of shapes with Miras also. Share and discuss.
- Provide examples of 2-D shapes with one line of symmetry, two lines of symmetry and no lines of symmetry. Have the students draw the lines of symmetry, sort the shapes and explain their thinking.
- Provide examples of 3 by 3 squares on grid paper. Shade 3 small squares so that the figure has one line of symmetry. Challenge the students to make as many different patterns with one line of symmetry by shading in three small squares or make figures with more than two lines of symmetry.



**Possible Models:** geoboards, Miras, pattern blocks, 2-D shapes, geometric dot paper, grid paper

SCO: **SS5: Demonstrate an understanding of line symmetry by:**

- identifying symmetrical 2-D shapes
- creating symmetrical 2-D shapes
- drawing one or more lines of symmetry in a 2-D shape.

[C, CN, V]

**SS6: Demonstrate an understanding of congruency, concretely and pictorially.**

[C, CN, V]

## ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

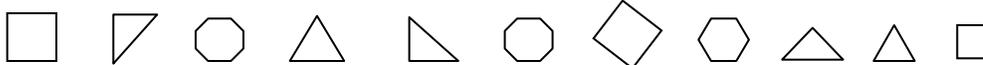
### Guiding Questions

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### Whole Class/Group/Individual Assessment

- Ask the student to name different 2-D shapes, to state how many lines of symmetry each has, and to show where the lines of symmetry are.
- Provide the diagrams of 2-D shapes some of which are congruent, such as the following:



Ask the students to: - put a check mark on shapes that are congruent to 

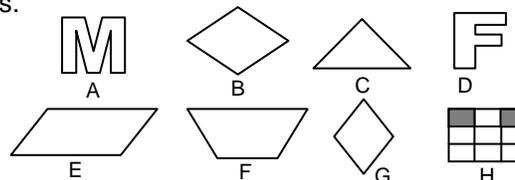
- put an X on shapes that are congruent to 

- shade in the shape that is congruent to 

Have the students explain the strategy they used to determine if the shapes were congruent.

Suggest that they trace and cut out the three shapes and then superimpose them on the given shapes to prove congruency.

- Give three examples of symmetrical shapes in your everyday world.
- Explain how congruent shapes are part of symmetrical shapes.
- Place the following labelled 2-D shapes before the student.



Have the student circle all the symmetrical shapes. Then instruct the student to draw all the lines of symmetry on the symmetrical shapes. Finally, have the student sort the shapes by the number of lines of symmetry in each shape: no lines of symmetry; one line of symmetry; more than 1 line of symmetry.

## FOLLOW-UP ON ASSESSMENT

### Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: <b>SP1: Demonstrate an understanding of many-to-one correspondence.</b> [C, R, T, V]			
[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<b>SP1</b> Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions. <b>SP2</b> Construct, label and interpret bar graphs to solve problems.	<b>SP1</b> Demonstrate an understanding of many-to-one correspondence.	<b>SP2</b> Construct and interpret double bar graphs to draw conclusions.

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

Prior to grade four, students have had opportunities to collect and display data in **pictographs** and **bar graphs**. As students investigate a wider range of topics, they may discover that the data they collect is too large to display in a graph using a one-to-one correspondence (i.e., a symbol or number on the bar graph represent one piece of data). Students need to be introduced to the concept of using a **many-to-one correspondence** (i.e., a **scale** that allows a single symbol to represent a number of items) when they are creating graphs to display large amounts of data. In grade four, students should begin to make decisions about what symbol to use and what that symbol should represent. These decisions are based on the data being used.

Students need to be given many opportunities to explore and choose what scale is most appropriate for their set of data. For example, if they want to display a graph to show their marble collection and they have 36 blue, 28 red, and 42 black, students may decide to draw symbols to each represent 5 marbles or create a scale in a bar graph that increases by 2. If the numbers are all less than 20, it is usually more appropriate to use a one-to-one correspondence. For larger numbers, however, students may find it better to use **intervals** (increments) of 2, 5, 10, 25, 100, or 1000 based on the data being graphed. Students should discuss their data displays and be able to explain why they chose their scale. Students would not be expected to use the term, 'interval' in their explanations, but may justify their choice by telling how they 'skip counted'. It is important for students to ensure that the interval in their data display is consistent. For example, if they are creating a bar graph that has a scale of two, all of the numbers need to increase by 2 (2, 4, 6, 8, 10, 12 ... and not 2, 4, 6, 7, 8, 9, 10, 12...). Depending on the data and the scale that is selected, it may become necessary to create partial pictograph symbols and bars that fall between numbers on the scale.

As students compare their own graphs and those from other sources, they should examine how the graphs are similar and different. Students should discuss why they think the interval or correspondence was chosen and what other scales may have also been used. Deciding on what scale to use requires students to apply their knowledge of multiplication and therefore, it is very helpful for students to have a good knowledge of these facts.

SCO: **SP1: Demonstrate an understanding of many-to-one correspondence.**  
[C, R, T, V]

### **ACHIEVEMENT INDICATORS**

#### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Compare graphs in which different intervals or correspondences are used and explain why the interval or correspondence was used.
- Compare graphs in which the same data has been displayed using one-to-one and many-to-one correspondences, and explain how they are the same and different.
- Explain why many-to-one correspondence is sometimes used rather than one-to-one correspondence.
- Find examples of graphs in which many-to-one correspondence is used in print and electronic media, such as newspapers, magazines and the Internet, and describe the correspondence used.

SCO: SP1: Demonstrate an understanding of many-to-one correspondence.  
[C, R, T, V]

## **PLANNING FOR INSTRUCTION**

Before introducing new material, consider ways to assess and build on students' knowledge and skills related to numbers.

### **Guiding Questions**

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

### **Choosing Instructional Strategies**

Consider the following strategies when planning lessons:

- Ensure that students are aware that scales typically start at zero.
- Allow students to decide on the scale for their data displays, but ensure that they can justify their choice.
- Have students work with a variety of sets of data, so they will have experiences creating different scales.
- Use data display software or websites for students to quickly create and compare graphs that have different scales.

### **Suggested Activities**

- Have the student redraw a pictograph so that each symbol represents 4, instead of 2. Ask the student which graph he/she prefers and to give reasons for the choice. Ask if there is another way to display the data which might be clearer.
- Provide data for a bar graph: such as Favourite Sports (hockey: 36, baseball: 20, basketball: 15, soccer: 26). Have the student select a scale and create a bar graph.
- Ask the students to determine the scale for a bar graph to display the number of students travelling on each different school bus in the morning. Each step along a bar is to represent more than one student.
- Pose a question such as the following: How much television do grade four students watch? Have students estimate about how many hours of television (or video games or computer time) they have watched in a week. Have students construct two pictographs for the same data. The intervals in one can be constructed using one-to-one correspondence and the other using many-to-one correspondence (e.g., ☺ = 5 hours). Have students explain which of the two graphs they prefer. Students should justify their preference.
- Have students explore other applications of many-to-one correspondence, such as the use of scale in mapping.
- Investigate the importance of using a consistent scale. Present the following graph and ask students if there are more girls or more boys watching soccer. Discuss why the graph is misleading.

Girls and Boys Watching a Soccer Game



**Possible Models:** grid paper, various collections of objects

SCO: **SP1: Demonstrate an understanding of many-to-one correspondence.**  
[C, R, T, V]

## **ASSESSMENT STRATEGIES**

Look back at what you determined as acceptable evidence.

### **Guiding Questions**

- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### **Whole Class/Group/One-on-One Assessment**

- Ask why a symbol in a pictograph usually represents more than 1.
- Provide students with two graphs: one that displays one-to-one correspondence and the other displays many-to-one correspondence. Explain the similarities and differences.
- Provide students with a set of data with large numbers and have them create a scale and a graph to display it. Ask the student to justify their choice of scale and graph.
- Ask the student for an example of when it would be appropriate to use a one-to-one correspondence using a real-life context.
- Ask the student for an example of when it would be more appropriate to use a many-to-one correspondence using a real-life context.

## **FOLLOW-UP ON ASSESSMENT**

### **Guiding Questions**

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**

[C, PS, R, V]

[C] Communication  
[T] Technology

[PS] Problem Solving  
[V] Visualization

[CN] Connections  
[R] Reasoning

[ME] Mental Math  
and Estimation

### Scope and Sequence of Outcomes

Grade Three	Grade Four	Grade Five
<p><b>SP1</b> Collect first-hand data and organize it using: tally marks; line plots; charts; lists to answer questions.</p> <p><b>SP2</b> Construct, label and interpret bar graphs to solve problems.</p>	<p><b>SP2</b> Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.</p>	<p><b>SP2</b> Construct and interpret double bar graphs to draw conclusions.</p>

### ELABORATION

#### Guiding Questions:

- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

The goal of this outcome is to help students recognize that graphs provide information and that different types of representations tell different things about the data. The value of having students actually construct their own graphs is not so much that they learn the techniques, but that they are personally invested in the data and that they learn how a graph conveys information. Once a graph is constructed, the most important activity is discussing what it tells the people who see it, especially those who were not involved in making the graph. Discussions about graphs of real data that students have themselves been involved in gathering will help them interpret other graphs and charts that they see in newspapers, on television and other media (Van de Walle & Lovin, vol. 2, 2006, p. 329).

Students extend their understanding of constructing graphs and interpreting data from previous grades by exploring **vertical** and **horizontal** displays that require a many-to-one correspondence. When creating **pictographs** and **bar graphs**, it is important for their displays include a **title**, **labels**, and a **legend** or **key** (when applicable).

Once students have constructed a graph, it is important for students to have an opportunity to make observations and interpret the data. They should also be given experiences discussing other graphs that they can find, such as in newspapers and magazines, and on television and the Internet. Questioning should be ongoing whenever students use graphs to encourage students to interpret the data presented and to draw inferences. It is important to ask questions that go beyond simplistic reading of a graph. Both literal questions and inferential questions should be posed. For example:

- How many ....?
- How many more/less than....?
- Order from least to greatest/ greatest to least...
- Based on the information presented in the graph, what other conclusions can you make?
- Why do you think . . . ?

Have students discuss what kinds of information they can get from reading bar graphs and pictographs that display the use of many-to-one correspondence.

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**  
[C, PS, R, V]

## **ACHIEVEMENT INDICATORS**

### **Guiding Questions:**

- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

Use the following set of indicators as a guide to determine whether students have met the corresponding specific outcome.

- Identify an interval (scale) and correspondence (one-to-one or many-to-one) for displaying a given set of data in a graph and justify the choice.
- Create and label (with categories, title and legend) a pictograph to display a given set of data using many-to-one correspondence, and justify the choice of correspondence (value of symbol) used.
- Create and label (with axes and title) a bar graph to display a given set of data using many-to-one correspondence, and justify the choice of interval (scale) used.
- Answer a given question using a given graph in which data is displayed using many-to-one correspondence.

SCO: SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

[C, PS, R, V]

## PLANNING FOR INSTRUCTION

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

### Guiding Questions

- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

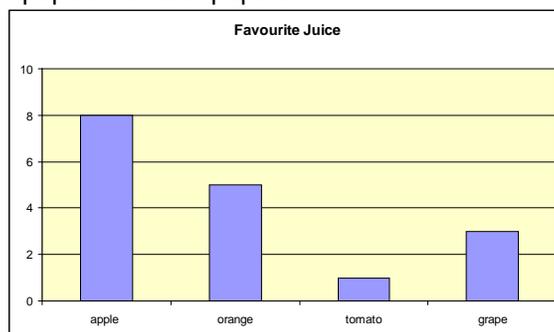
### Choosing Instructional Strategies

Consider the following strategies when planning lessons:

- Use pictographs based on a many-to-one correspondence (i.e., a symbol represents a group of items) and bar graphs that have intervals of more than one (i.e., increments of 2, 5, 10, 25, 100, etc.).
- Provide students with a variety of experiences to ensure that when creating bar graphs and pictographs, students have an understanding of the importance of including a title and labels and using appropriate intervals (scales) and correspondence for their data.
- Have students interpret and create various bar graphs and pictographs that run horizontally and vertically.
- Create graphs primarily in the context of other investigations, including other subject areas, rather than as an isolated activity to achieve the curriculum outcome.
- Allow opportunities for students to decide on which scales to use for their graphs.
- Help students to investigate that many questions can be answered by looking at graphs.

### Suggested Activities

- Suggest that students create a graph that shows the most popular authors, movies, types of food, etc. of class members. Have some students create a bar graph that shows the results of data in scale of 2 and other groups can use a scale of 3, 4, and 5. Have students explain which graph displays the most appropriate use of the data.
- Show a graph like the one below. Explain that the spacing between each horizontal line represents 2 students and ask questions, such as: "How many students like apple juice? How many more like apple juice than tomato juice? How many students answered the questions about their favourite juice?" Order the juices from most popular to least popular.



- Have students discuss what kinds of information they can get from reading existing bar graphs and pictographs that display the use of many-to-one correspondence.

**Possible Models:** pre-made pictographs, pre-made bar graphs, grid paper

SCO: **SP2: Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.**  
[C, PS, R, V]

## ASSESSMENT STRATEGIES

Look back at what you determined as acceptable evidence.

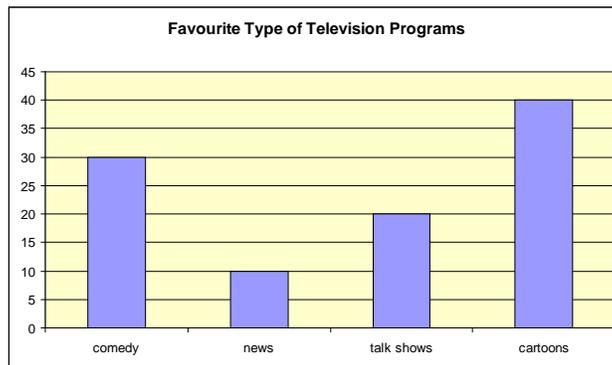
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Assessment can and should happen every day as a part of instruction. A variety of approaches and contexts should be used for assessing all students: as a class, in groups, and individual students. Consider the following **sample activities** (that can be adapted) for either formative (for learning; as learning) or summative (of learning) assessment.

### Whole Class/Group/Individual Assessment

- Ask students what questions might be answered by interpreting this graph?



- Create and label (with categories, title and legend) a pictograph and bar graph using the table below about “Favourite Movies” using many-to-one correspondence, and justify the choice of scale or correspondence used.

Adventure	29
Comedy	28
Drama	25
Science Fiction	35

- Throughout the year provide opportunities for students to self-assess their graphs. Here are some suggestions for students to complete:
  - I know I constructed a good graph because...
  - Some things that are similar between my graph and my classmate’s graph are...
  - Some things that are different about my graph and my classmate’s graph are...
  - When I make a graph I choose intervals of 2 (or 5 or 10, etc.) when...
  - When I make a graph, I choose to use an interval of 1 when...

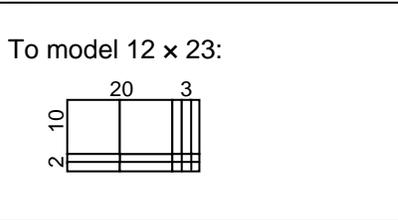
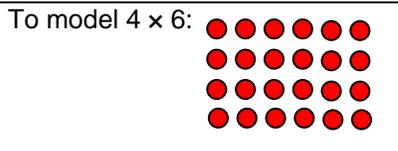
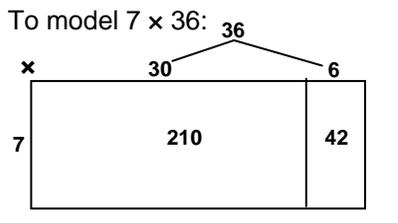
## FOLLOW-UP ON ASSESSMENT

### Guiding Questions

- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

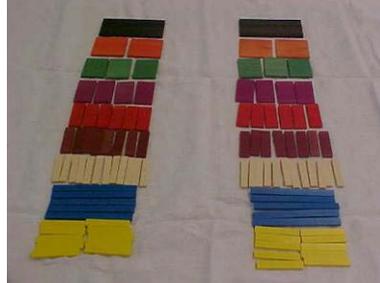
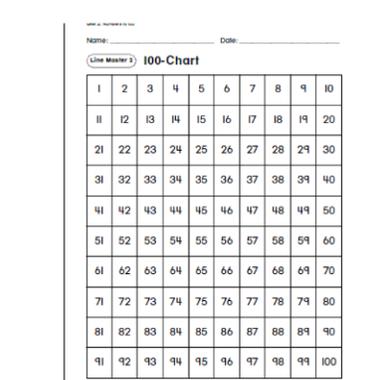
### GLOSSARY OF MODELS

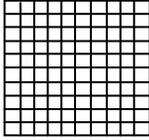
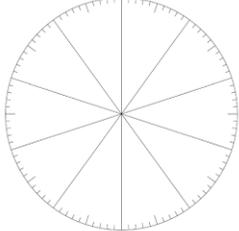
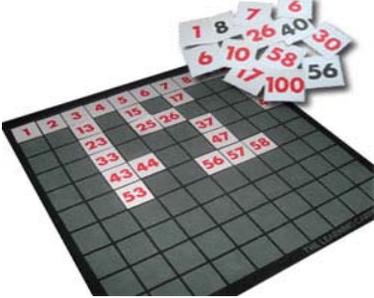
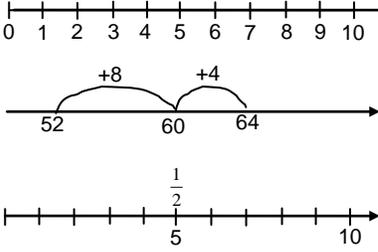
This glossary is identical for all grade levels (kindergarten to grade 8). Most of the models have a variety of uses at different grade levels. More information as to which models can be used to develop specific curriculum outcomes is located on the *Instructional Strategies* section of each four-page spread in this curriculum document. The purpose of this glossary is to provide a visual of each model and a brief description of it.

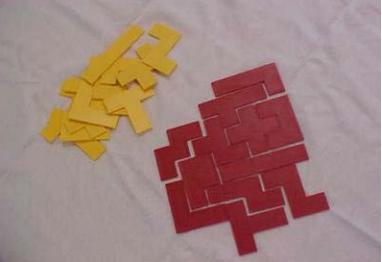
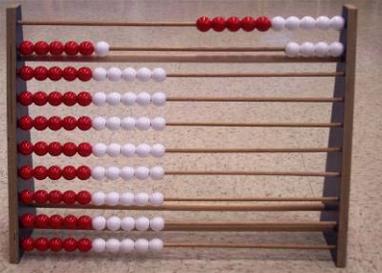
Name	Picture	Description
<b>Algebra tiles</b>		<ul style="list-style-type: none"> <li>Sets include “X” tiles (rectangles), “X<sup>2</sup>” tiles (large squares), and integer tiles (small squares).</li> <li>All tiles have a different colour on each side to represent positive and negative. Typically the “X” tiles are green and white and the smaller squares are red and white.</li> <li>Some sets also include “Y” sets of tiles which are a different colour and size than the “X” tiles.</li> </ul>
<b>Area Model</b>	<p>To model <math>12 \times 23</math>:</p> 	<ul style="list-style-type: none"> <li>Use base ten blocks to represent the parts of each number that is being multiplied.</li> <li>To find the answer for the example shown, students can add the various parts of the model: <math>200 + 30 + 40 + 6 = 276</math>.</li> <li>This model can also be used for fraction multiplication.</li> </ul>
<b>Arrays and Open Arrays</b>	<p>To model <math>4 \times 6</math>:</p>  <p>To model <math>7 \times 36</math>:</p> 	<ul style="list-style-type: none"> <li>Use counters arranged in equal rows or columns or a Blackline Master with rows and columns of dots.</li> <li>Helpful in developing understanding of multiplication facts.</li> <li>Grids can also be used to model arrays.</li> <li>Open arrays allows students to think in amounts that are comfortable for them and does not lock them into thinking using a specific amount. These arrays help visualize repeated addition and partitioning and ultimately using the distributive property.</li> </ul>
<b>Attribute Blocks</b>		<ul style="list-style-type: none"> <li>Sets of blocks that vary in their attributes:             <ul style="list-style-type: none"> <li>5 shapes circle, triangle, square, hexagon, rectangle</li> <li>2 thicknesses</li> <li>2 sizes</li> <li>3 colours</li> </ul> </li> </ul>
<b>Balance (pan or beam) scales</b>		<ul style="list-style-type: none"> <li>Available in a variety of styles and precision.</li> <li>Pan balances have a pan or platform on each side to compare two unknown amounts or represent equality. Weights can be used on one side to measure in standard units.</li> <li>Beam balances have parallel beams with a piece that is moved on each beam to determine the mass of the object on the scale. Offer greater accuracy than a pan balance.</li> </ul>

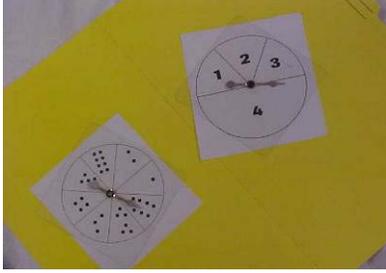
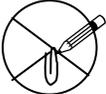
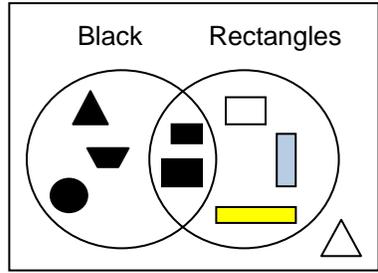
<p><b>Base Ten Blocks</b></p>		<ul style="list-style-type: none"> <li>• Include unit cubes, rods, flats, and large cubes.</li> <li>• Available in a variety of colours and materials (plastic, wood, foam).</li> <li>• Usually 3-D.</li> </ul>									
<p><b>Beam Balance</b></p>	<p>☞ see Balance (pan or beam)</p>										
<p><b>Carroll Diagram</b></p>	<p>Example:</p> <table border="1" data-bbox="431 594 813 688"> <tr> <td></td> <td>1-digit</td> <td>2-digit</td> </tr> <tr> <td>Even</td> <td>2, 4, 6, 8</td> <td>26, 34</td> </tr> <tr> <td>Odd</td> <td>1, 3, 5, 7, 9</td> <td>15, 21</td> </tr> </table>		1-digit	2-digit	Even	2, 4, 6, 8	26, 34	Odd	1, 3, 5, 7, 9	15, 21	<ul style="list-style-type: none"> <li>• Used for classification of different attributes.</li> <li>• The table shows the four possible combinations for the two attributes.</li> <li>• Similar to a Venn Diagram</li> </ul>
	1-digit	2-digit									
Even	2, 4, 6, 8	26, 34									
Odd	1, 3, 5, 7, 9	15, 21									
<p><b>Colour Tiles</b></p>		<ul style="list-style-type: none"> <li>• Square tiles in 4 colours (red, yellow, green, blue).</li> <li>• Available in a variety of materials (plastic, wood, foam).</li> </ul>									
<p><b>Counters (two colour)</b></p>		<ul style="list-style-type: none"> <li>• Counters have a different colour on each side.</li> <li>• Available in a variety of colour combinations, but usually are red &amp; white or red &amp; yellow.</li> <li>• Available in different shapes (circles, squares, bean).</li> </ul>									
<p><b>Cubes (Linking)</b></p>		<ul style="list-style-type: none"> <li>• Set of interlocking 2 cm cubes.</li> <li>• Most connect on all sides.</li> <li>• Available in a wide variety of colours (usually 10 colours in each set).</li> <li>• Brand names include: Multilink, Hex-a-Link, Cube-A-Link.</li> <li>• Some types only connect on two sides (brand name example: Unifix).</li> </ul>									
<p><b>Cuisenaire Rods®</b></p>		<ul style="list-style-type: none"> <li>• Set includes 10 different colours of rods.</li> <li>• Each colour represents a different length and can represent different number values or units of measurement.</li> <li>• Usual set includes 74 rods (22 white, 12 red, 10 light green, 6 purple, 4 yellow, 4 dark green, 4 black, 4 brown, 4 blue, 4 orange).</li> <li>• Available in plastic or wood.</li> </ul>									

<p><b>Dice (Number Cubes)</b></p>		<ul style="list-style-type: none"> <li>• Standard type is a cube with numbers or dots from 1 to 6 (number cubes).</li> <li>• Cubes can have different symbols or words.</li> <li>• Also available in:             <ul style="list-style-type: none"> <li>○ 4-sided (tetrahedral dice)</li> <li>○ 8-sided (octahedral dice)</li> <li>○ 10-sided (decahedra dice)</li> <li>○ 12-sided, 20-sided, and higher</li> <li>○ Place value dice</li> </ul> </li> </ul>
<p><b>Dominoes</b></p>		<ul style="list-style-type: none"> <li>• Rectangular tiles divided in two-halves.</li> <li>• Each half shows a number of dots: 0 to 6 or 0 to 9.</li> <li>• Sets include tiles with all the possible number combinations for that set.</li> <li>• Double-six sets include 28 dominoes.</li> <li>• Double-nine sets include 56 dominoes.</li> </ul>
<p><b>Dot Cards</b></p>		<ul style="list-style-type: none"> <li>• Sets of cards that display different number of dots (1 to 10) in a variety of arrangements.</li> <li>• Available as free Blackline Master online on the "Teaching Student-Centered Mathematics K-3" website (BLM 3-8).</li> </ul>
<p><b>Decimal Squares®</b></p>		<ul style="list-style-type: none"> <li>• Tenths and hundredths grids that are manufactured with parts of the grids shaded.</li> <li>• Can substitute a Blackline Master and create your own class set.</li> </ul>
<p><b>Double Number Line</b></p>	<p>☞ see Number lines (standard, open, and double)</p>	
<p><b>Five-frames</b></p>	<p>☞ see Frames (five- and ten-)</p>	
<p><b>Fraction Blocks</b></p>		<ul style="list-style-type: none"> <li>• Also known as Fraction Pattern blocks.</li> <li>• 4 types available: pink "double hexagon", black chevron, brown trapezoid, and purple triangle.</li> <li>• Use with basic pattern blocks to help study a wider range of denominators and fraction computation.</li> </ul>
<p><b>Fraction Circles</b></p>		<ul style="list-style-type: none"> <li>• Sets can include these fraction pieces:             <math display="block">1, \frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}</math> </li> <li>• Each fraction graduation has its own colour.</li> <li>• It is helpful to use ones without the fractions marked on the pieces for greater flexibility (using different piece to represent 1 whole).</li> </ul>

<p><b>Fraction Pieces</b></p>		<ul style="list-style-type: none"> <li>Rectangular pieces that can be used to represent the following fractions:  <math display="block">\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}</math> </li> <li>Offers more flexibility as different pieces can be used to represent 1 whole.</li> <li>Each fraction graduation has its own colour.</li> <li>Sets available in different quantities of pieces.</li> </ul>
<p><b>Frames (five- and ten-)</b></p>		<ul style="list-style-type: none"> <li>Available as a Blackline Master in many resources or you can create your own.</li> <li>Use with any type of counter to fill in the frame as needed.</li> </ul>
<p><b>Geoboards</b></p>		<ul style="list-style-type: none"> <li>Available in a variety of sizes and styles.                         <ul style="list-style-type: none"> <li>5 x 5 pins</li> <li>11 x 11 pins</li> <li>Circular 24 pin</li> <li>Isometric</li> </ul> </li> <li>Clear plastic models can be used by teachers and students on an overhead.</li> <li>Some models can be linked to increase the size of the grid.</li> </ul>
<p><b>Geometric Solids</b></p>		<ul style="list-style-type: none"> <li>Sets typically include a variety of prisms, pyramids, cones, cylinders, and spheres.</li> <li>The number of pieces in a set will vary.</li> <li>Available in different materials (wood, plastic, foam) and different sizes.</li> </ul>
<p><b>Geo-strips</b></p>		<ul style="list-style-type: none"> <li>Plastic strips that can be fastened together with brass fasteners to form a variety of angles and geometric shapes.</li> <li>Strips come in 5 different lengths. Each length is a different colour.</li> </ul>
<p><b>Hundred Chart</b></p>		<ul style="list-style-type: none"> <li>10 x 10 grid filled in with numbers 1-100 or 0 - 99.</li> <li>Available as a Blackline Master in many resources or you can create your own.</li> <li>Also available as wall charts or "Pocket" charts where cards with the numbers can be inserted or removed.</li> </ul>

<p><b>Hundred Grid</b></p>		<ul style="list-style-type: none"> <li>• 10 × 10 grid.</li> <li>• Available as Blackline Master in many resources.</li> </ul>
<p><b>Hundredths Circle</b></p>	<p>Percent Circles</p> 	<ul style="list-style-type: none"> <li>• Circle divided into tenths and hundredths.</li> <li>• Also known as “percent circles”.</li> </ul>
<p><b>Learning Carpet®</b></p>	 <p><a href="http://www.thelearningcarpet.ca">http://www.thelearningcarpet.ca</a></p>	<ul style="list-style-type: none"> <li>• 10 × 10 grid printed on a floor rug that is six feet square.</li> <li>• Number cards and other accessories are available to use with the carpet.</li> </ul>
<p><b>Linking Cubes</b></p>	<p>☞ see Cubes (Linking)</p>	
<p><b>Mira®</b></p>		<ul style="list-style-type: none"> <li>• Clear red plastic with a bevelled edge that projects reflected image on the other side.</li> <li>• Other brand names include: Reflect-View and Math-Vu™.</li> </ul>
<p><b>Number Cubes</b></p>	<p>☞ see Dice (Number Cubes)</p>	
<p><b>Number Lines (standard, open, and double)</b></p>		<ul style="list-style-type: none"> <li>• Number lines can begin at 0 or extend in both directions.</li> <li>• Open number lines do not include pre-marked numbers or divisions. Students place these as needed.</li> <li>• Double number lines have numbers written above and below the line to show equivalence.</li> </ul>
<p><b>Open Arrays</b></p>	<p>☞ see Arrays and Open Arrays</p>	
<p><b>Open Number Lines</b></p>	<p>☞ see Number Lines (standard, open, and double)</p>	
<p><b>Pan Balance</b></p>	<p>☞ see Balance (pan or beam)</p>	

<b>Pattern Blocks</b>		<ul style="list-style-type: none"> <li>• Standard set includes: Yellow hexagons, red trapezoids, blue parallelograms, green triangles, orange squares, beige parallelograms.</li> <li>• Available in a variety of materials (wood, plastic, foam).</li> </ul>
<b>Pentominoes</b>		<ul style="list-style-type: none"> <li>• Set includes 12 unique polygons.</li> <li>• Each is composed of 5 squares which share at least one side.</li> <li>• Available in 2-D and 3-D in a variety of colours.</li> </ul>
<b>Polydrons</b>		<ul style="list-style-type: none"> <li>• Geometric pieces snap together to build various geometric solids as well as their nets.</li> <li>• Pieces are available in a variety of shapes, colours, and sizes: Equilateral triangles, isosceles triangles, right-angle triangles, squares, rectangles, pentagons, hexagons</li> <li>• Also available as Frameworks (open centres) that work with Polydrons and another brand called G-O-Frames™.</li> </ul>
<b>Power Polygons™</b>		<ul style="list-style-type: none"> <li>• Set includes the 6 basic pattern block shapes plus 9 related shapes.</li> <li>• Shapes are identified by letter and colour.</li> </ul>
<b>Rekenrek</b>		<ul style="list-style-type: none"> <li>• Counting frame that has 10 beads on each bar: 5 white and 5 red.</li> <li>• Available with different number of bars (1, 2, or 10).</li> </ul>

<p><b>Spinners</b></p>		<ul style="list-style-type: none"> <li>• Create your own or use manufactured ones that are available in a wide variety:             <ul style="list-style-type: none"> <li>○ number of sections;</li> <li>○ colours or numbers;</li> <li>○ different size sections;</li> <li>○ blank.</li> </ul> </li> <li>• Simple and effective version can be made with a pencil held at the centre of the spinner with a paperclip as the part that spins.</li> </ul> 
<p><b>Tangrams</b></p>		<ul style="list-style-type: none"> <li>• Set of 7 shapes (commonly plastic):             <ul style="list-style-type: none"> <li>○ 2 large right-angle triangles</li> <li>○ 1 medium right-angle triangle</li> <li>○ 2 small right-angle triangles</li> <li>○ 1 parallelogram</li> <li>○ 1 square</li> </ul> </li> <li>• 7-pieces form a square as well as a number of other shapes.</li> <li>• Templates also available to make sets.</li> </ul>
<p><b>Ten-frames</b></p>	<p>☞ see Frames (five- and ten-)</p>	
<p><b>Trundle Wheel</b></p>		<ul style="list-style-type: none"> <li>• Tool for measuring longer distances.</li> <li>• Each revolution equals 1 metre usually noted with a click.</li> </ul>
<p><b>Two Colour Counters</b></p>	<p>☞ see Counters (two colour)</p>	
<p><b>Venn Diagram</b></p>		<ul style="list-style-type: none"> <li>• Used for classification of different attributes.</li> <li>• Can be one, two, or three circles depending on the number of attributes being considered.</li> <li>• Attributes that are common to each group are placed in the interlocking section.</li> <li>• Attributes that don't belong are placed outside of the circle(s), but inside the rectangle.</li> <li>• Be sure to draw a rectangle around the circle(s) to show the "universe" of all items being sorted.</li> <li>• Similar to a Carroll Diagram.</li> </ul>

## List of Grade 4 Specific Curriculum Outcomes

### **Number (N)**

1. Represent and describe whole numbers to 10 000, pictorially and symbolically.
2. Compare and order numbers to 10 000.
3. Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3 and 4-digit numerals).
4. Explain the properties of 0 and 1 for multiplication and the property of 1 for division.
5. Describe and apply mental mathematics strategies, such as: skip counting from a known fact; using doubling or halving; using doubling or halving and adding or subtracting one more group; using patterns in the 9s facts; using repeated doubling to determine basic multiplication facts to  $9 \times 9$  and related division facts.
6. Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems.
7. Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: using personal strategies for dividing with and without concrete materials; estimating quotients; relating division to multiplication.
8. Demonstrate an understanding of fractions less than or equal to one by using concrete and pictorial representations to: name and record fractions for the parts of a whole or a set; compare and order fractions; model and explain that for different wholes, two identical fractions may not represent the same quantity; provide examples of where fractions are used.
9. Describe and represent decimals (tenths and hundredths) concretely, pictorially and symbolically.
10. Relate decimals to fractions (to hundredths).
11. Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by: using compatible numbers; estimating sums and differences; using mental math strategies to solve problems.

### **Patterns & Relations (PR)**

#### **(Patterns)**

1. Identify and describe patterns found in tables and charts, including a multiplication chart.
2. Reproduce a pattern shown in a table or chart using concrete materials.
3. Represent and describe patterns and relationships using charts and tables to solve problems.
4. Identify and explain mathematical relationships using charts and diagrams to solve problems.

#### **(Variables and Equations)**

5. Express a given problem as an equation in which a symbol is used to represent an unknown number.
6. Solve one-step equations involving a symbol to represent an unknown number.

### **Shape and Space (SS)**

#### **(Measurement)**

1. Read and record time using digital and analog clocks, including 24-hour clocks.
2. Read and record calendar dates in a variety of formats.
3. Demonstrate an understanding of area of regular and irregular 2-D shapes by: recognizing that area is measured in square units selecting and justifying referents for the units  $\text{cm}^2$  or  $\text{m}^2$ ; estimating area by using referents for  $\text{cm}^2$  or  $\text{m}^2$ ; determining and recording area ( $\text{cm}^2$  or  $\text{m}^2$ ); constructing different rectangles for a given area ( $\text{cm}^2$  or  $\text{m}^2$ ) in order to demonstrate that many different rectangles may have the same area.

#### **(3-D Objects and 2-D Shapes)**

4. Describe and construct rectangular and triangular prisms.

#### **(Transformations)**

5. Demonstrate an understanding of line symmetry by: identifying symmetrical 2-D shapes; creating symmetrical 2-D shapes; drawing one or more lines of symmetry in a 2-D shape.
6. Demonstrate an understanding of congruency, concretely and pictorially.

### **Statistics and Probability (SP)**

#### **(Data Analysis)**

1. Demonstrate an understanding of many-to-one correspondence.
2. Construct and interpret pictographs and bar graphs involving many-to-one correspondence to draw conclusions.

#### **(Chance and Uncertainty)**

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